# Camera

## Computer Graphics

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*(Some of this slides are borrowed from Prof. Yung-Yu Chuang)*

# Outline

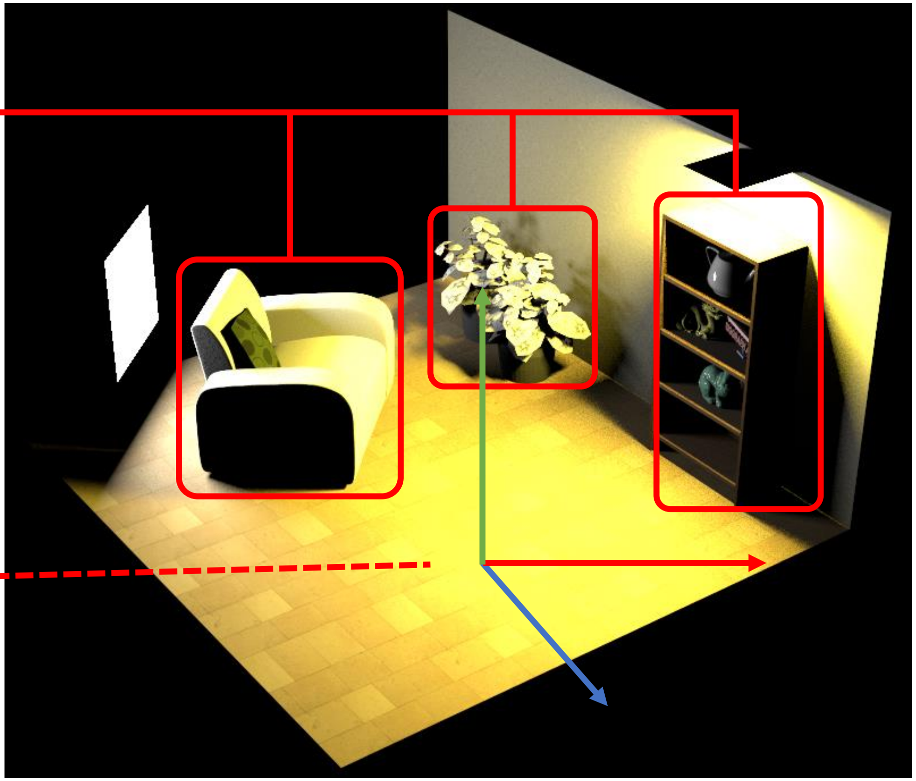
- Introduction to real-world cameras
- Introduction to computer graphics cameras
- Camera space and camera transformation
- Projective cameras
- OpenGL Implementation

# Recap.

- So far, we have introduced how to represent a virtual 3D world

Sofa, plant, bookshelf, and the room
vertex data ➔ (vertex buffer)
vertex adjacency ➔ (index buffer)
defined in **Object Space**

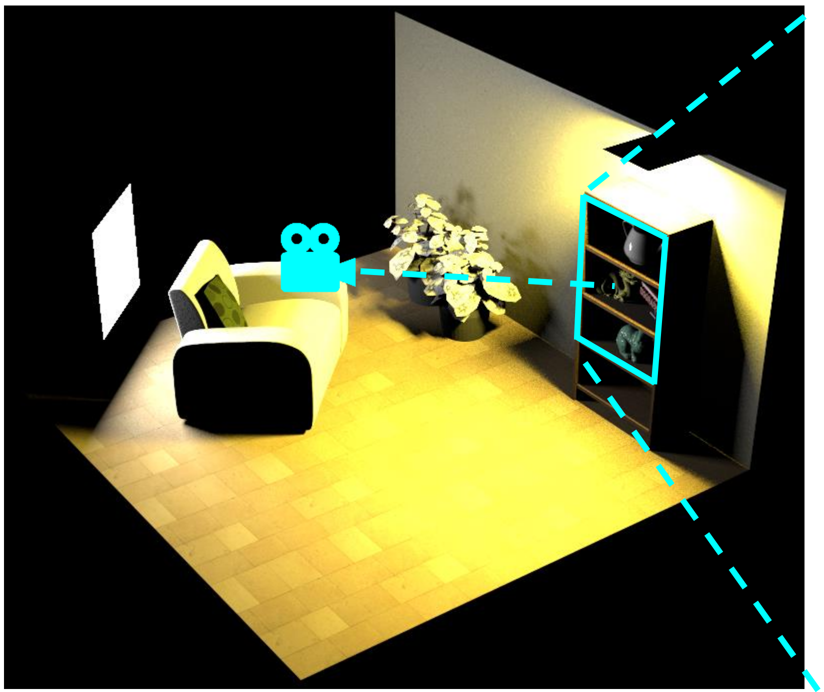
Objects are put into a shared
**World Space**
by **transformation**
**(translation, scaling, rotation)**



3D virtual world

# Recap. (cont.)

- In computer graphics, we generate an **image** from a **virtual 3D world** using a **virtual camera**

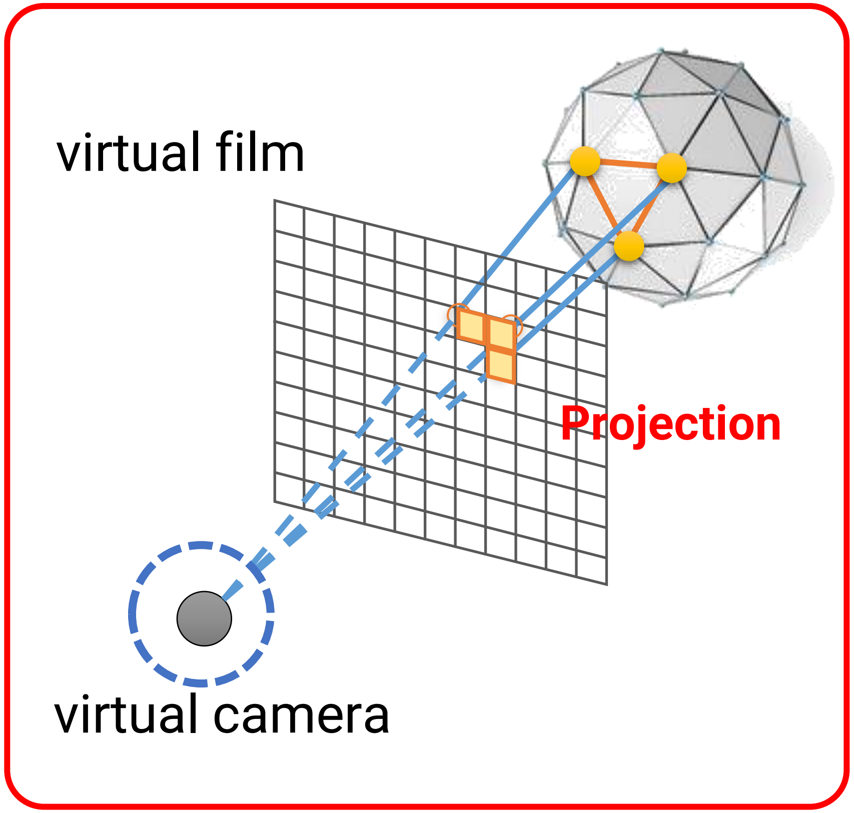


3D virtual world


rendered image

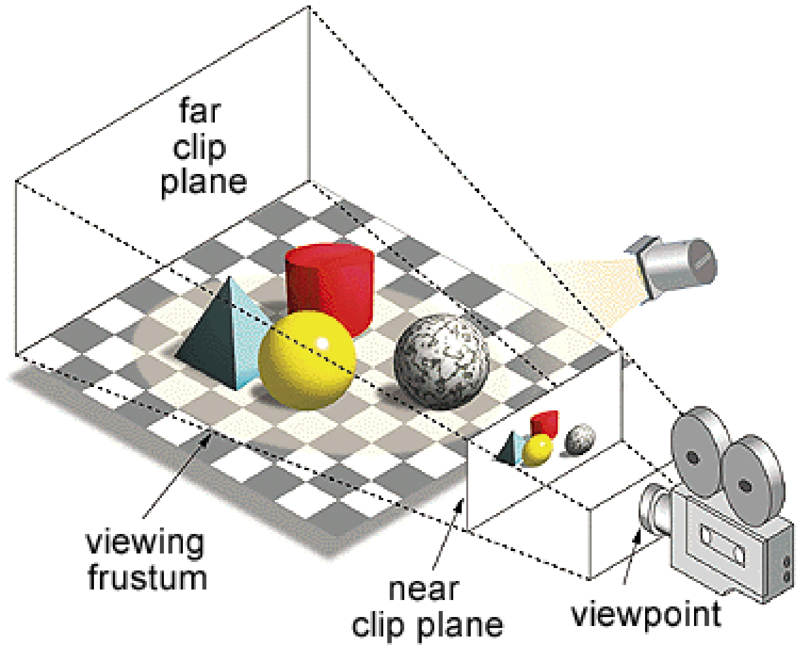
# Recap. (cont.)

- OpenGL uses **Rasterization** to bring 3D shapes to the 2D screen



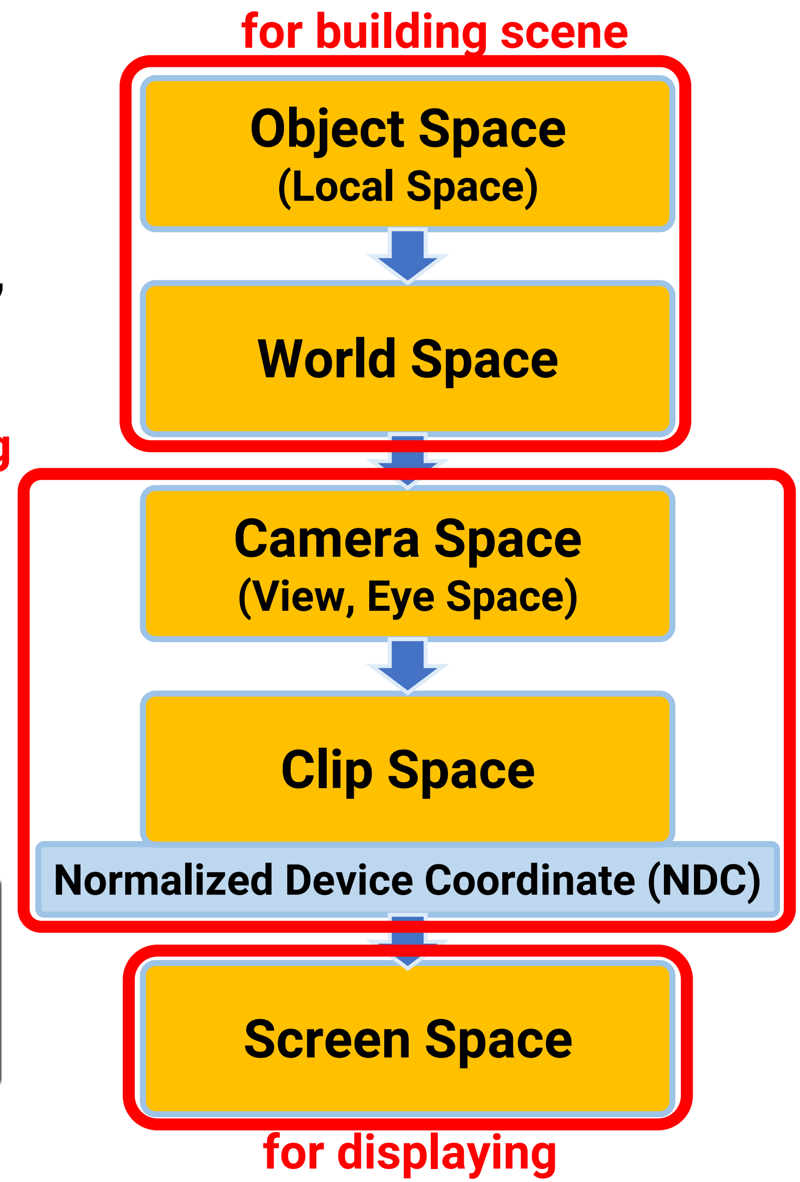
# Where is the Camera and Projection?

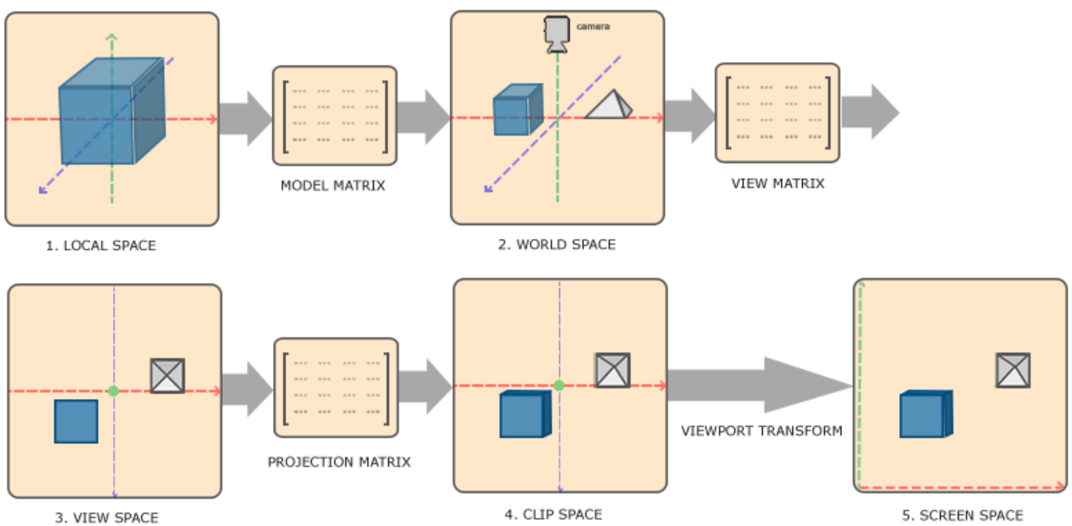
- The typical flow of bringing a 3D point to the 2D screen involves the **camera projection**
- For now, we specify neither the camera nor the projection, so you can consider that we set the **“projected”** positions of the vertices directly



# Spoiler

- There are other spaces
- We will introduce **camera space**, **clip space**, and **NDC** today

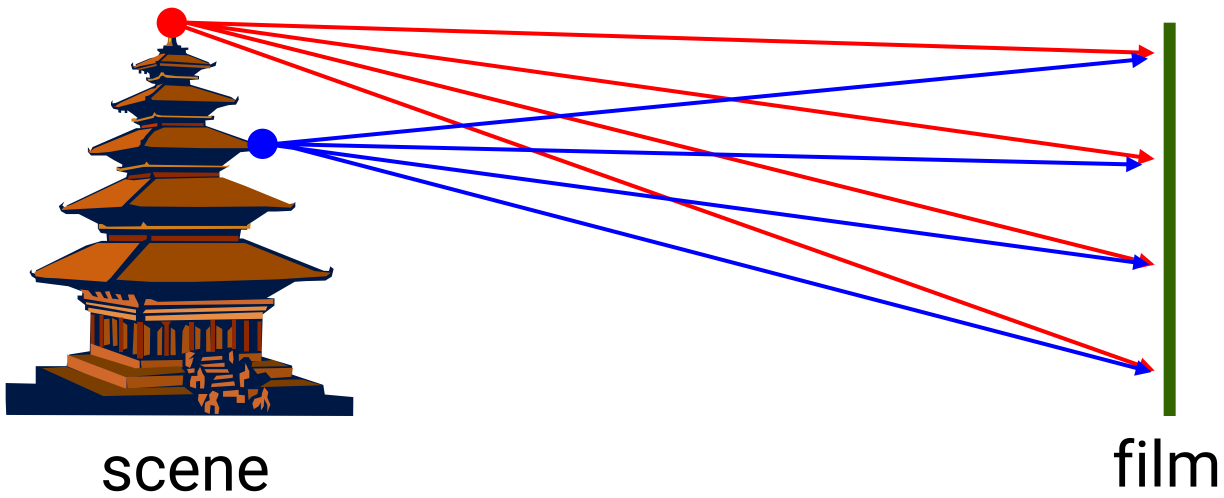


# Outline

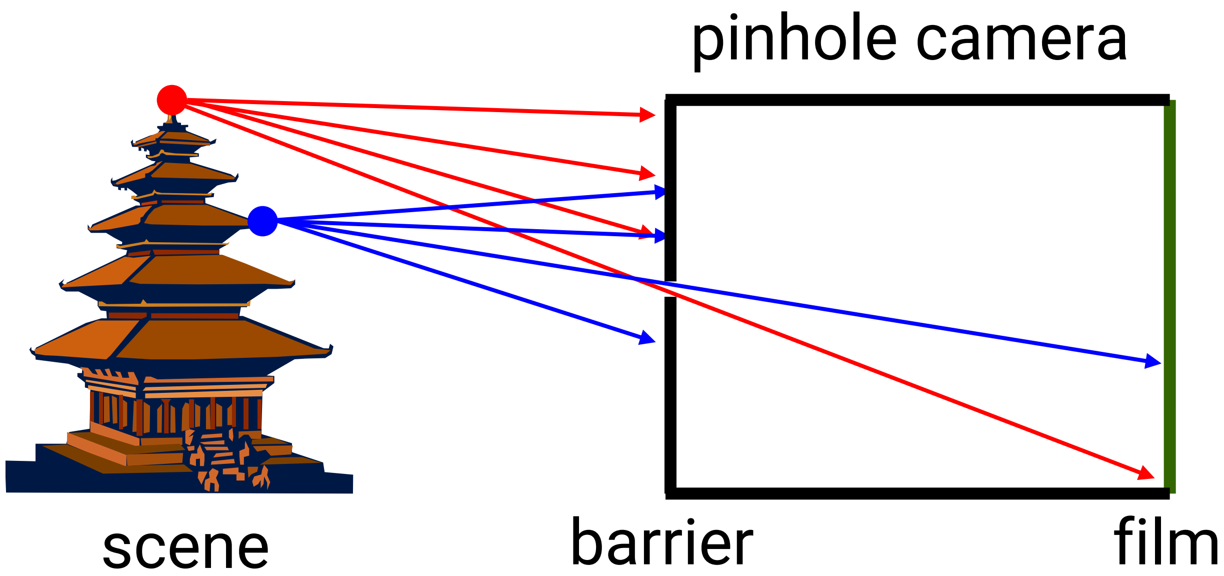
- **Introduction to real-world cameras**
- Introduction to computer graphics cameras
- Camera space and camera transformation
- Projective cameras
- OpenGL Implementation

# Camera Trail




Put a piece of film in front of an object

# Pinhole Camera

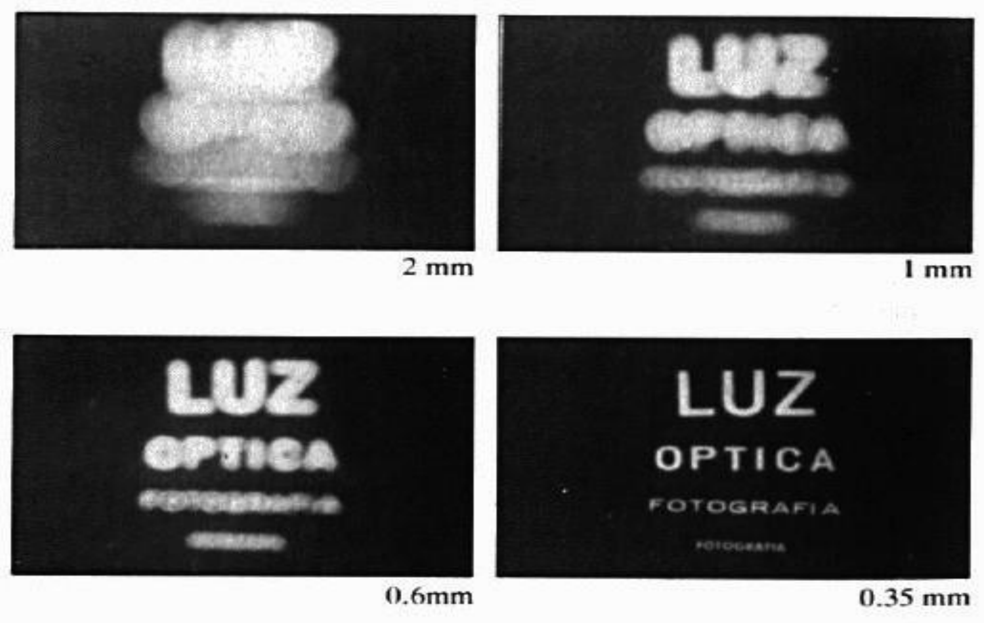



Add a barrier to block off most of the rays

- It reduces blurring
- The pinhole is known as the aperture
- The image is inverted

# Pinhole Camera (cont.)

- Shrink the aperture



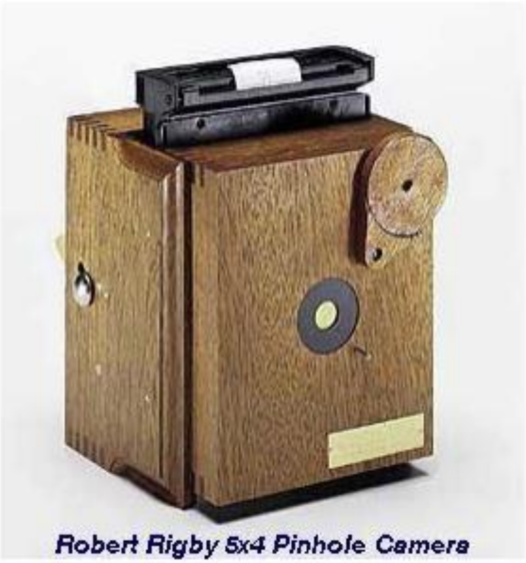

Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effect

# Pinhole Camera (cont.)

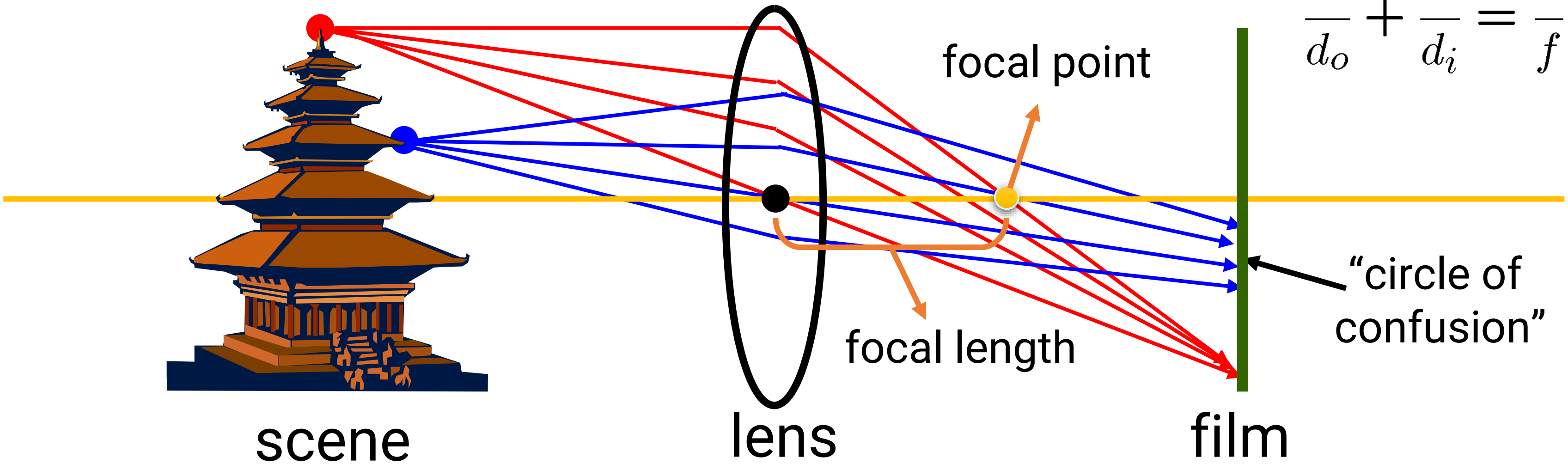
- Shrink the aperture

# Pinhole Camera (cont.)




$200~$700

# Camera with Lens




$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

A lens **focuses** light onto the film

- There is a specific distance at which objects are "in focus"
- Other points project to a "circle of confusion" in the image

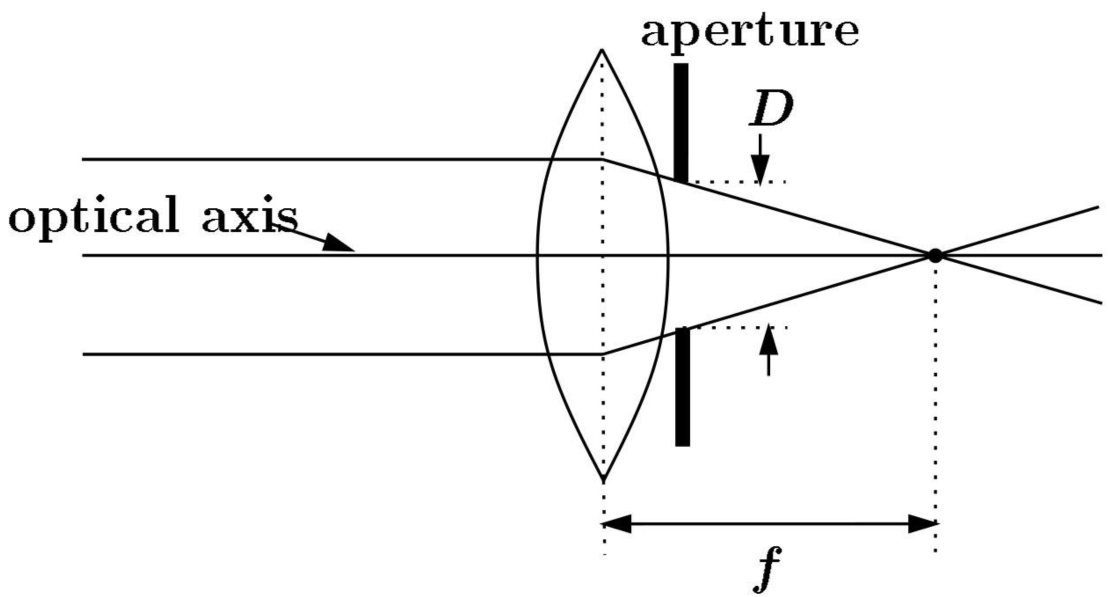
Current digital cameras replace the film with a **sensor array** (CCD or CMOS)

# Camera with Lens

Depth of field due to circle of confusion

# Exposure

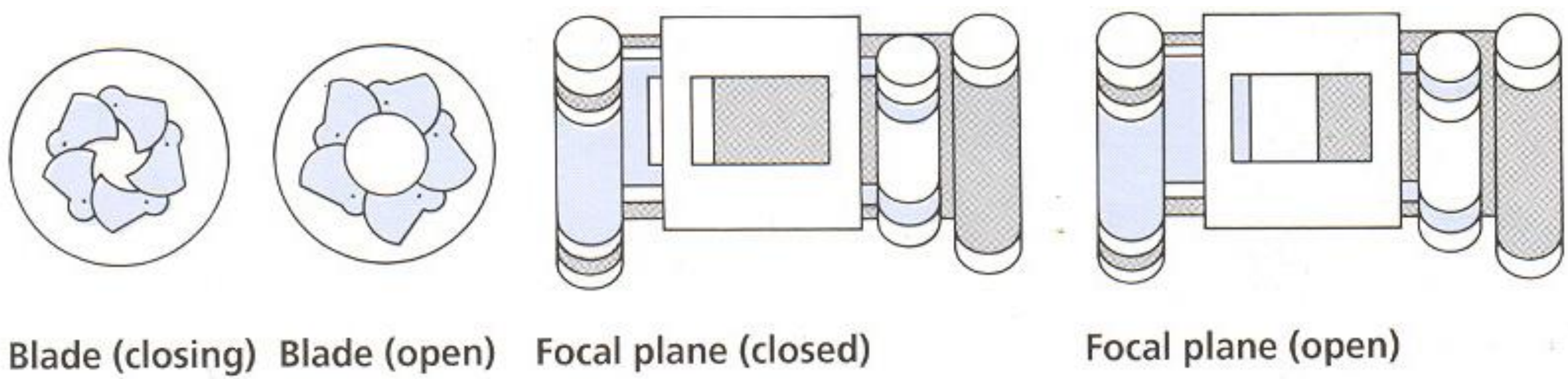
- **Exposure = aperture + shutter speed**
  - Aperture of diameter ***D*** restricts the range of rays (aperture may be on either side of the lens)
  - Shutter speed is the amount of time that light is allowed to pass through the aperture



# Exposure

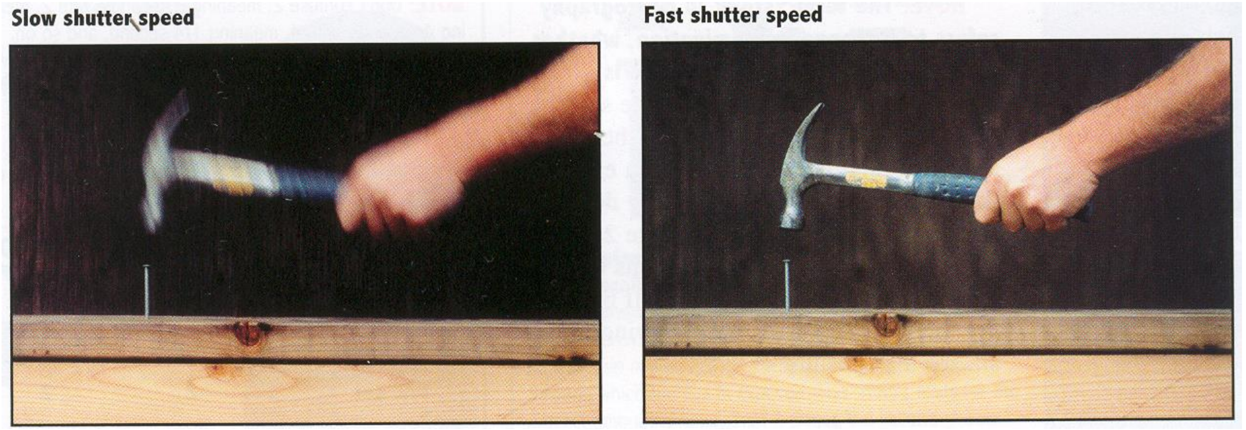
- Aperture (in f stop)


- Shutter speed (in fraction of a second)



# Motion Blur and Depth of Field

- **Motion blur**




- **Depth of field**

1/2500 sec at f / 1.8 | 1/500 sec at f / 4.0

# More About Real-World Cameras

- For more details about real-world cameras, please refer to the recording of my course, “Multimedia Technology and Application”
    - Course material link:
        - Part 1: https://reurl.cc/5dadyn
        - Part 2: https://reurl.cc/93y3Nn
        - Part 3: https://reurl.cc/A2D2mQ

# Outline

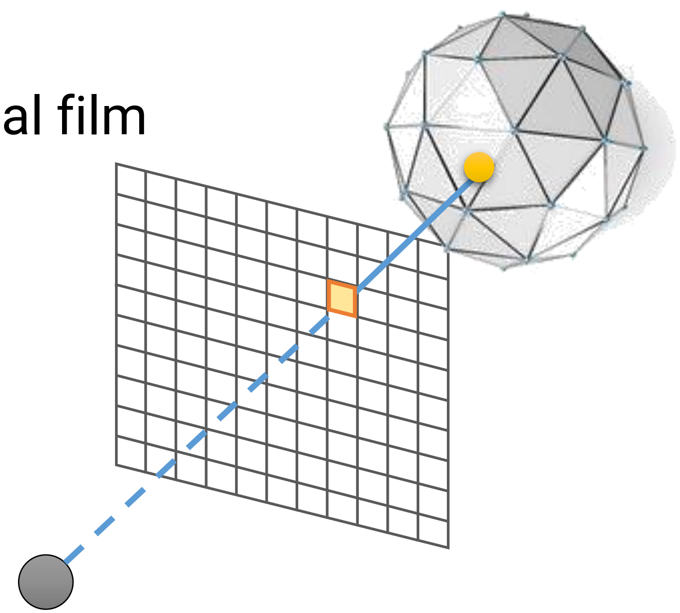
- Introduction to real-world cameras
- **Introduction to computer graphics cameras**
- Camera space and camera transformation
- Projective cameras
- OpenGL Implementation

# Recap. Again !

- Two ways for generating synthetic images

## Ray tracing

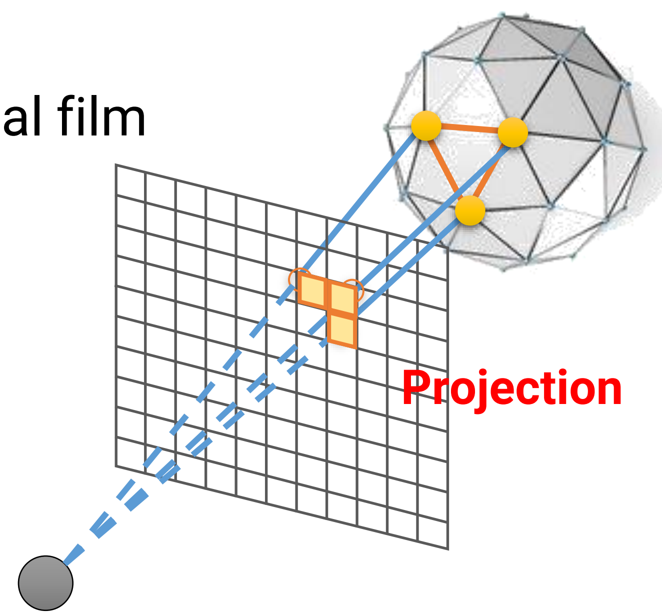
virtual film



virtual camera

## Rasterization

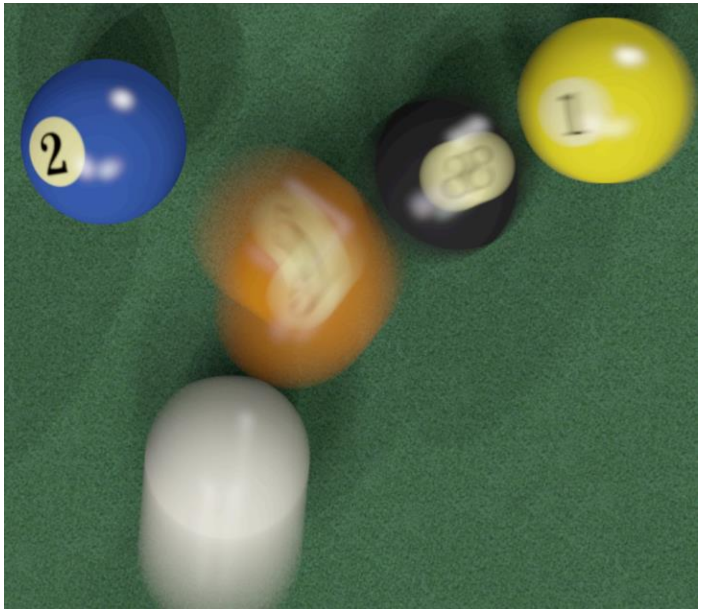
virtual film




virtual camera

# Computer Graphics Cameras

- Mimic the real-world functionalities of a real-world camera
- In offline (high-quality) graphics, we can simulate all the imaging processes of a camera using ray tracing

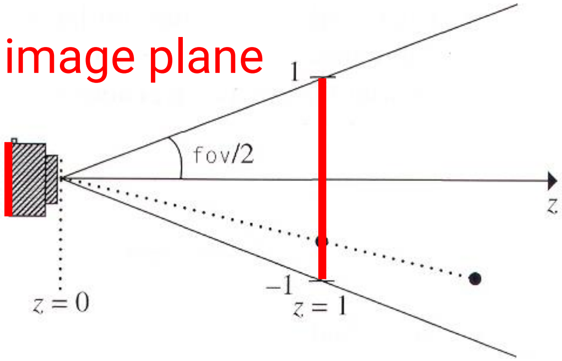


35 mm wide-angle

16 mm fisheye

# Computer Graphics Cameras (cont.)

- Mimic the real-world functionalities of a real-world camera
- In offline (high-quality) graphics, we can simulate all the imaging processes of a camera using ray tracing
- In interactive or real-time graphics, we usually use a **pinhole camera** for its simplicity of **projection**
- We can also dodge the drawbacks of real pinhole cameras
  - Avoid upside down images by **putting the film in front of the camera**

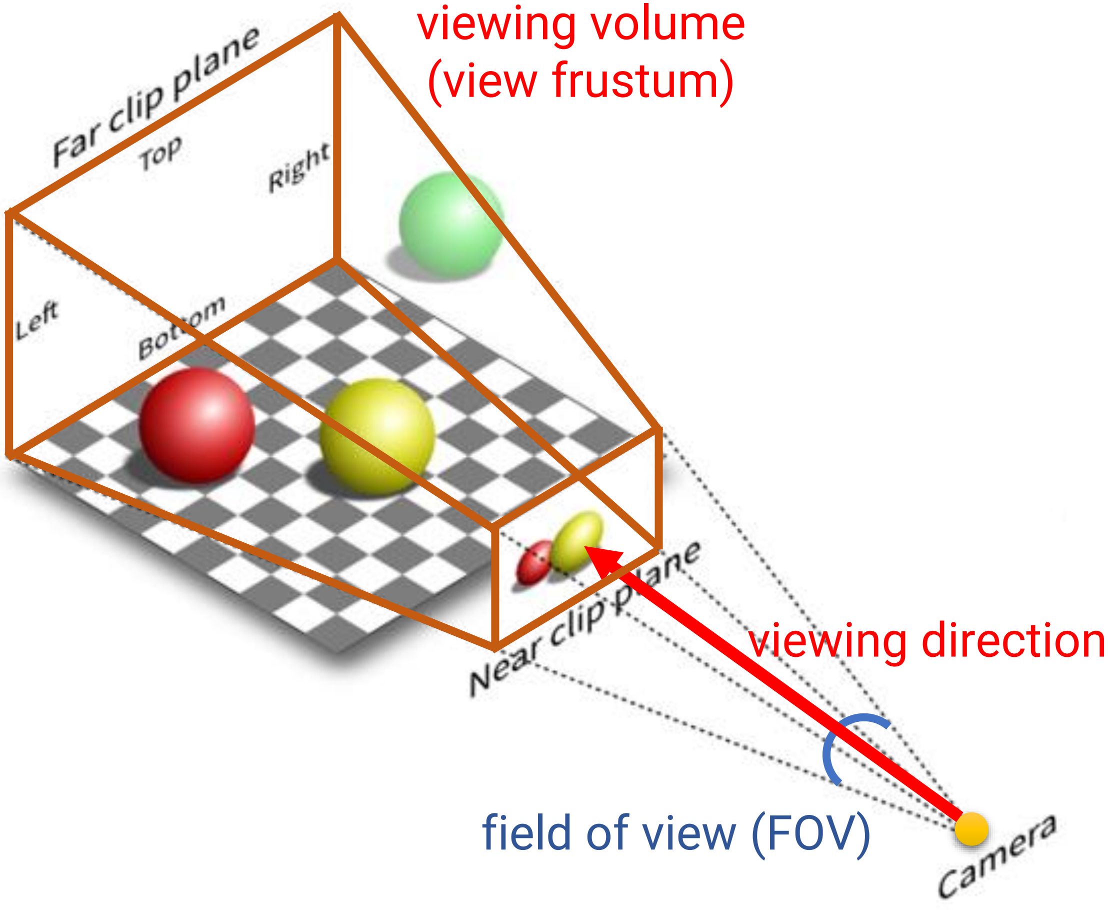


# Computer Graphics Camera (cont.)

- Every object will always be in focus

# Camera Properties

- Camera position
- Viewing direction
- Field of view
  - In angle
- Aspect ratio
  - Width/Height
- View volume
  - Near clip plane
  - Far clip plane



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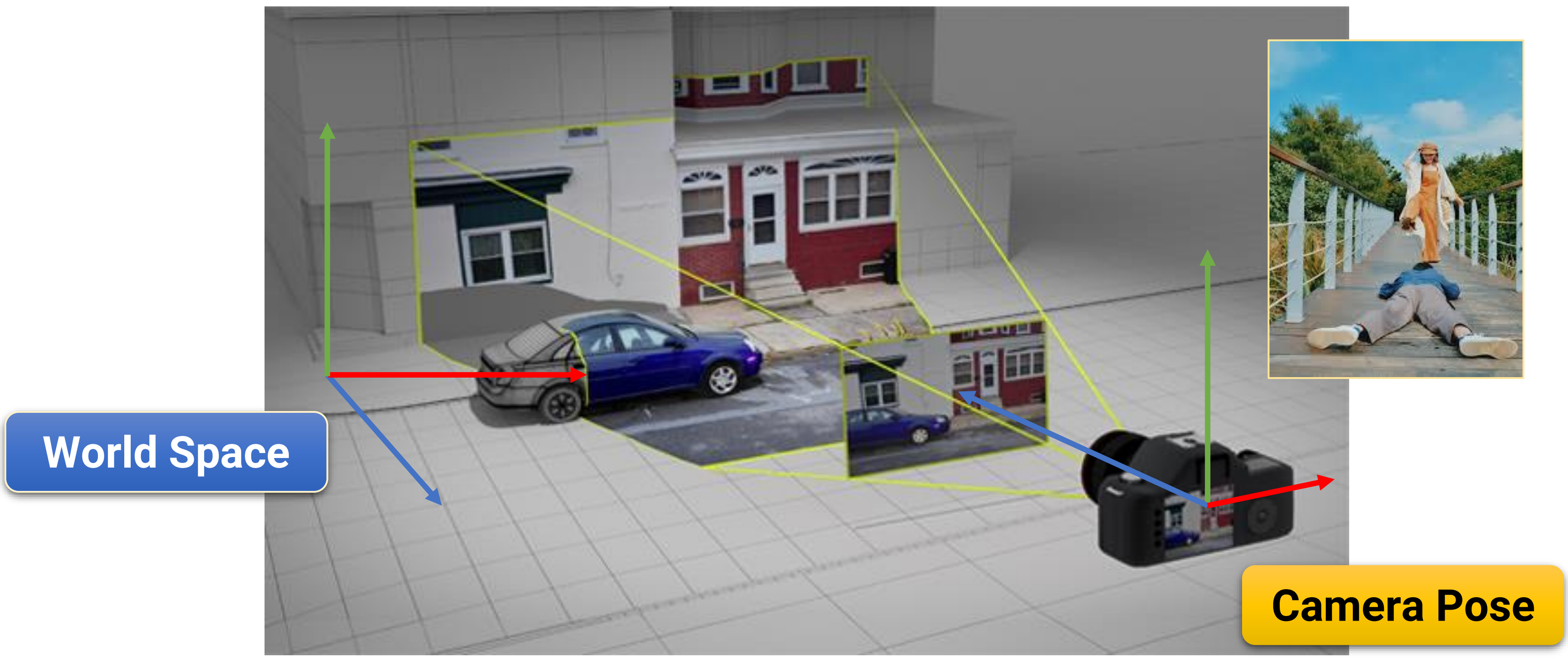
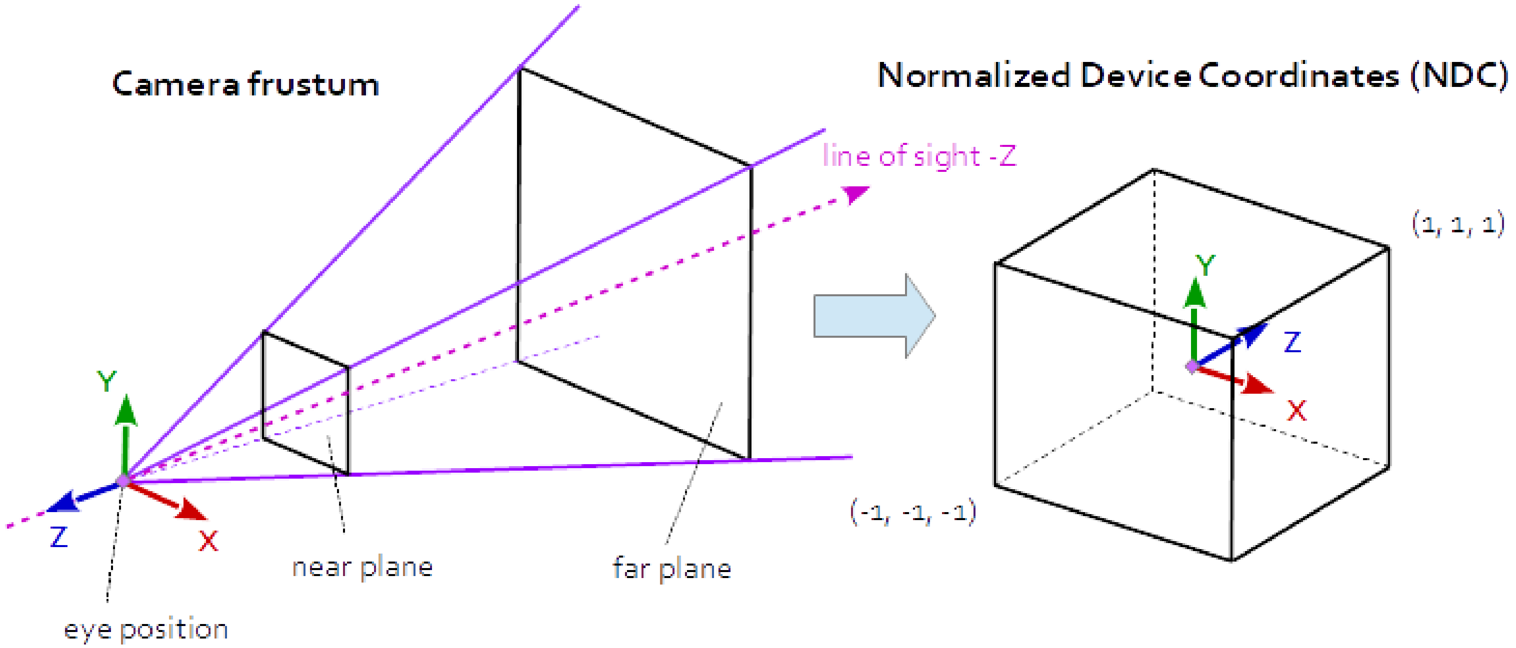
# Camera (View) Transform

- The camera can be at an arbitrary position and have an arbitrary viewing direction in the **world space**
- This makes the projection difficult in terms of mathematics



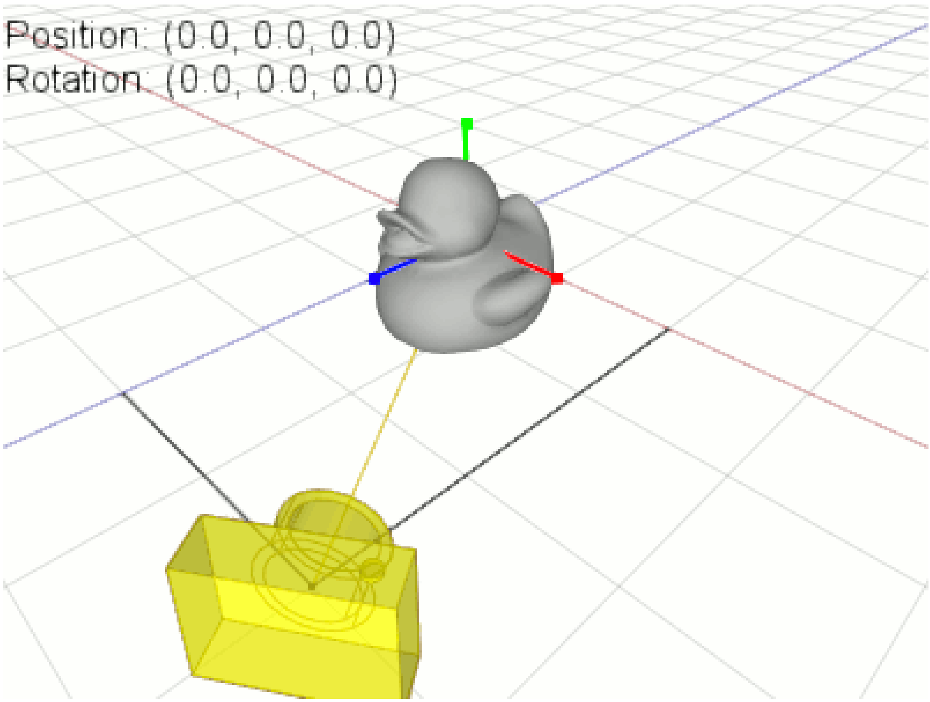
# Camera (View) Transform (cont.)

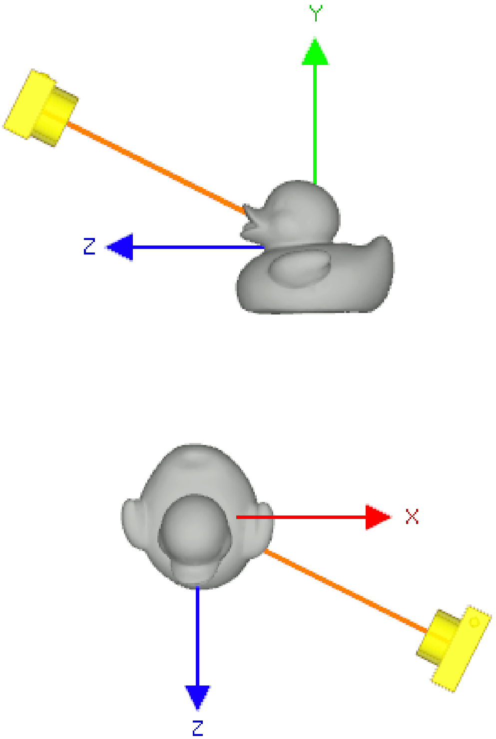
- To keep the math of projection simpler, we additionally define a **camera (view, eye) space**
  - In the camera space, the camera is **at the origin (0, 0, 0)** and **looking at the negative Z-axis**



# Camera (View) Transform (cont.)

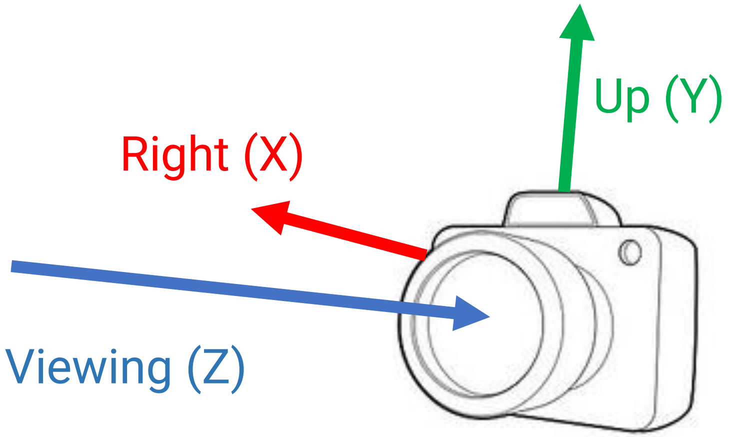
- OpenGL itself is not familiar with the concept of a camera
- Instead, we simulate one by moving all objects in the scene in the reverse direction

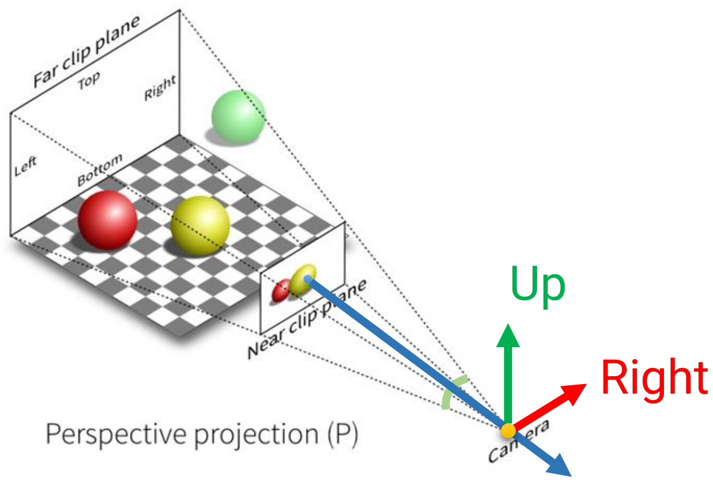


# Camera (View) Transform (cont.)

- For each object, we transform its world coordinate to the camera coordinate by
  - Moving it with the inverse translation of the camera's position
  - Rotate the object to match the **camera's local frame**




- Formed by the **view direction (D)**, **right (R)**, and **up (U)** vectors of the camera
- The three axes of the local frame should be **orthogonal**

# Camera (View) Transform (cont.)

- Set camera's local frame
  - However, it is usually difficult for a user to specify an orthogonal basis
  - OpenGL will do it for you (with the **Gram-Schmidt process**)

# Camera (View) Transform (cont.)

- Steps for setting camera's local frame
  - Determine the **viewing dir.** with the position of the camera and a target point

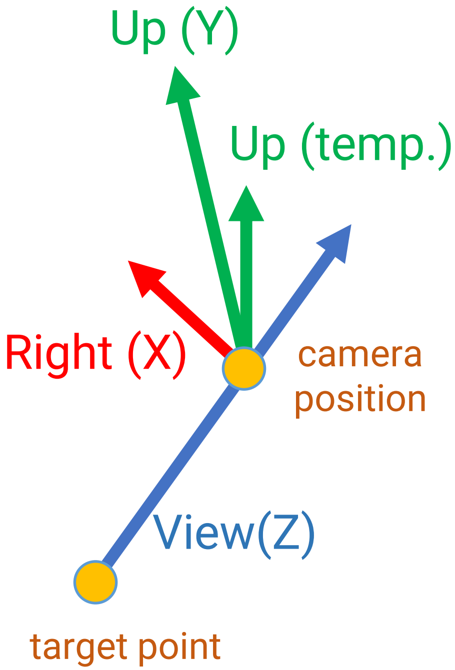
*viewing direction = normalize(cameraPos – targetPos)*

  - Assume a temporal "up vector"
    - In most cases, we use the up direction (0, 1, 0) in the world frame
  - Obtain the right vector by computing the **cross product** of the **up vector** and the **viewing dir.**

*camera right = normalize(cross(up, viewing direction))*

  - Obtain the **new up vector** by computing the **cross product** of the **viewing dir.** and the **right vector**

*camera up = normalize(cross( viewing direction, camera right))*



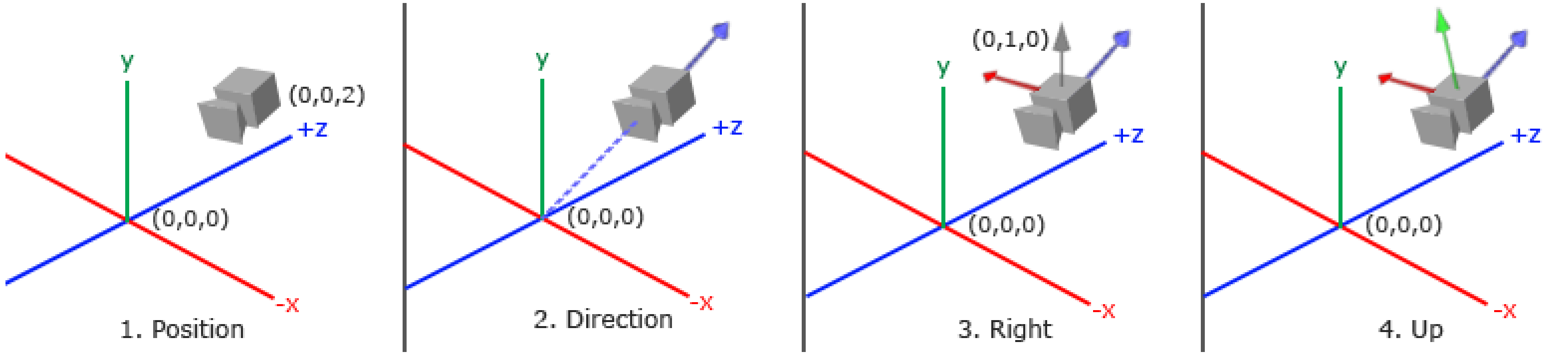
# Camera (View) Transform (cont.)

- Camera (view) transformation

($P_x$, $P_y$, $P_z$) is the camera's position

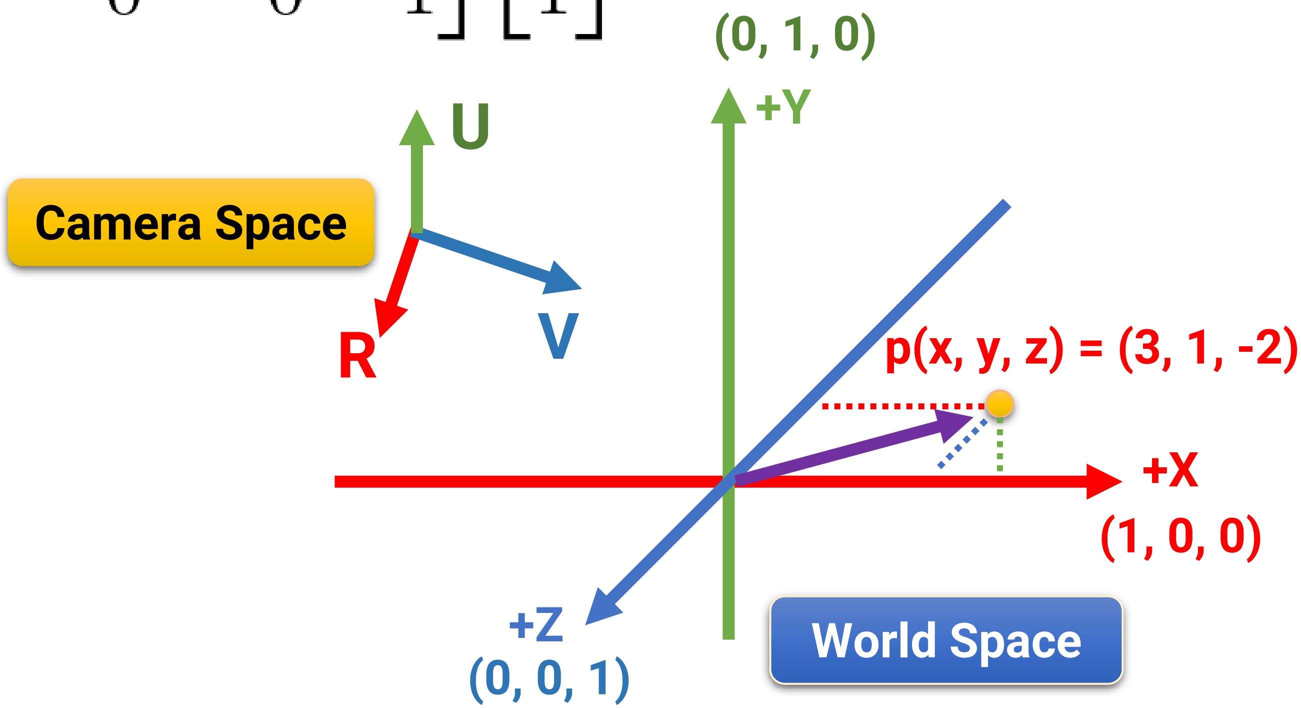
$$\begin{array}{r}\text{right vector}\\[4pt] \text{up vector}\\[4pt] \text{viewing vector}\\[4pt] \phantom{0}\end{array}\begin{bmatrix}R_x & R_y & R_z & 0\\ U_x & U_y & U_z & 0\\ D_x & D_y & D_z & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & -P_x\\ 0 & 1 & 0 & -P_y\\ 0 & 0 & 1 & -P_z\\ 0 & 0 & 0 & 1\end{bmatrix}$$

rotation matrix　　translation matrix



# Camera (View) Transform (cont.)

$$\begin{array}{r} \text{right vector} \\ \text{up vector} \\ \text{viewing vector} \\ \phantom{0} \end{array} \begin{bmatrix} R_x & R_y & R_z & 0 \\ U_x & U_y & U_z & 0 \\ D_x & D_y & D_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



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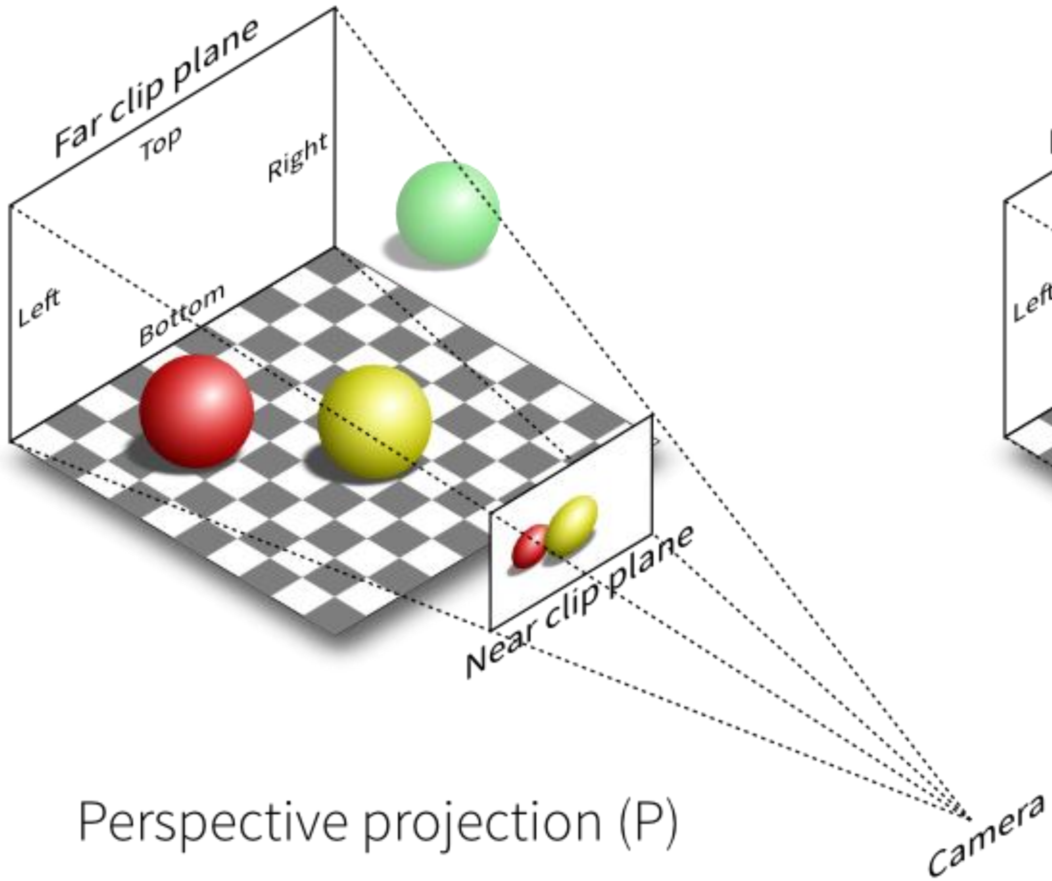
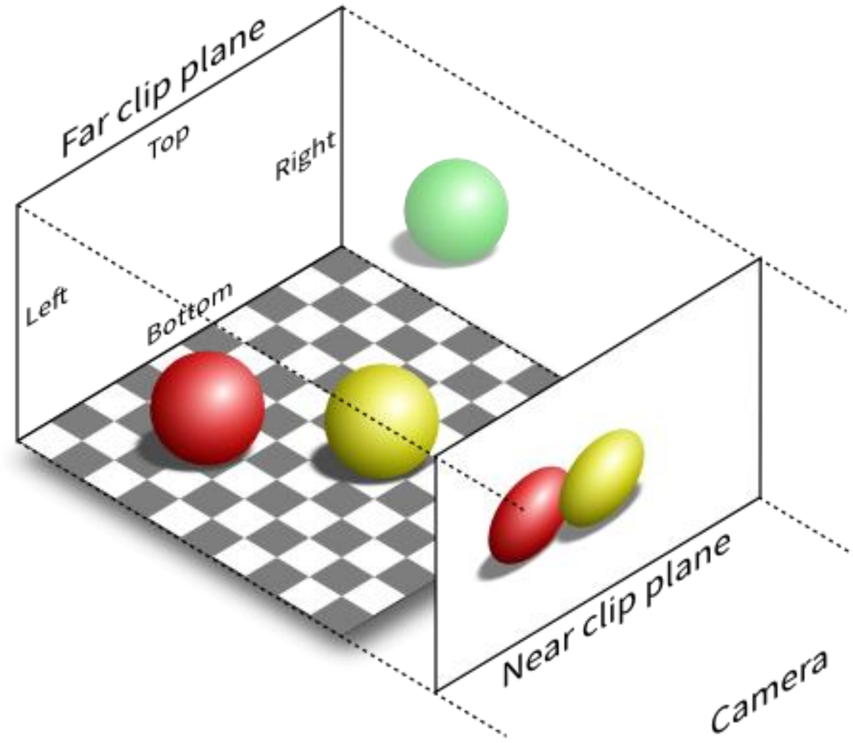
# Projective Camera Models



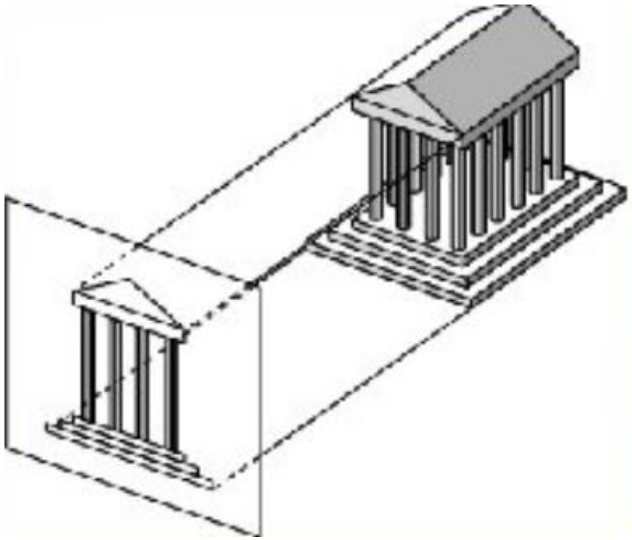

Perspective projection (P)

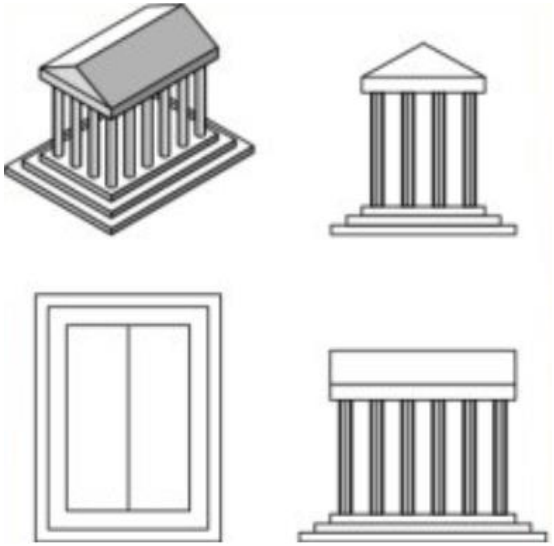



Orthographic projection (O)

# Orthographic Projection

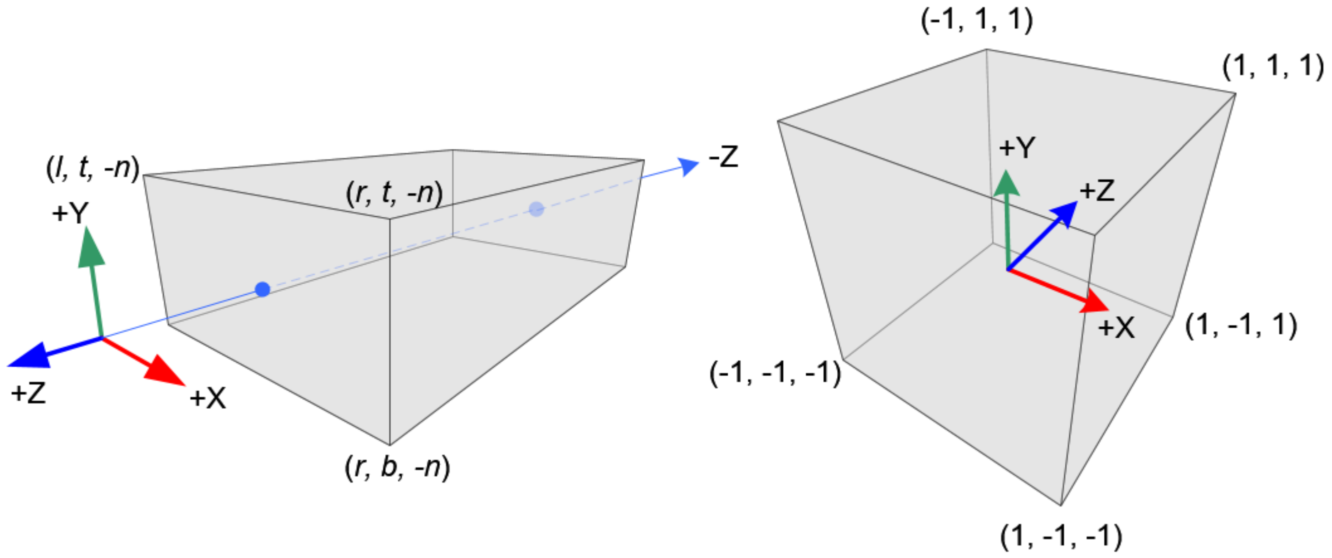
- Parallel projection with projectors perpendicular to the projection plane
- Preserve distance and angle
- Often used as front, side, and top views for 3D design





# Orthographic Projection (cont.)

- Need to define the viewing volume with its six planes: left, right, top, bottom, near, and far
  - The viewing volume (frustum) is cube-like
- Map the xyz-coordinate to the range [-1, 1]



# Orthographic Projection (cont.)

- Let the **l**, **r**, **t**, **b**, **n**, **f** be the boundaries of the left, right, top, bottom, near, and far planes

$$l \leq x \leq r \quad \Rightarrow \quad 0 \leq x - l \leq r - l$$

$$\Rightarrow \quad 0 \leq \frac{x - l}{r - l} \leq 1 \quad \Rightarrow \quad 0 \leq 2\left(\frac{x - l}{r - l}\right) \leq 2$$

$$\Rightarrow \quad -1 \leq 2\left(\frac{x - l}{r - l}\right) - 1 \leq 1 \quad \Rightarrow \quad -1 \leq \frac{2\boxed{x}}{r - l} - \frac{r + l}{r - l} \leq 1$$

# Orthographic Projection (cont.)

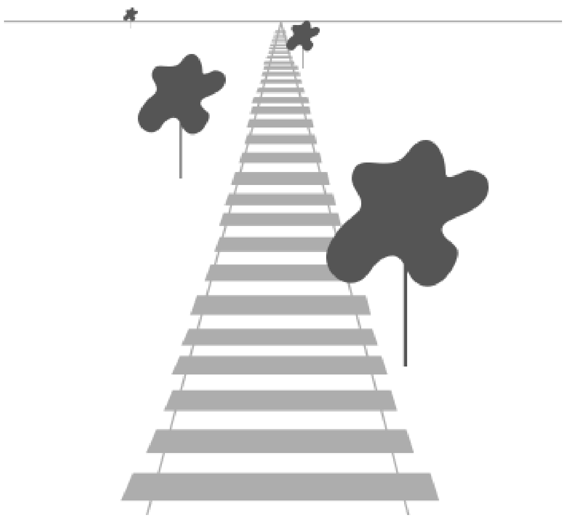
- Let the **l**, **r**, **t**, **b**, **n**, **f** be the boundaries of the left, right, top, bottom, near, and far planes
- An orthographic projection matrix can be written as

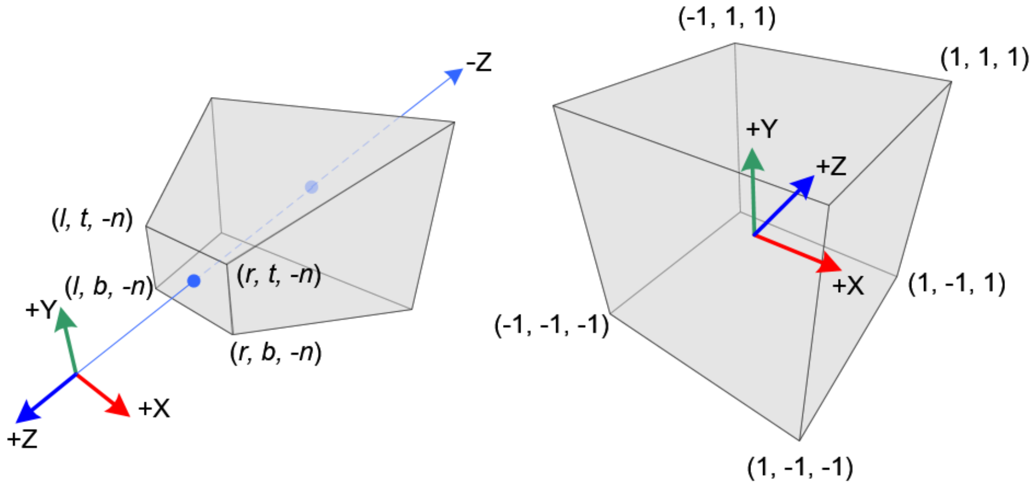
$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-1 \leq \frac{2x}{r-l} - \frac{r+l}{r-l} \leq 1$$

# Perspective Projection

- In our real lives, the objects that are farther away appear much smaller
- This effect is called **perspective**
- A perspective projection tries to mimic the vision of human eyes



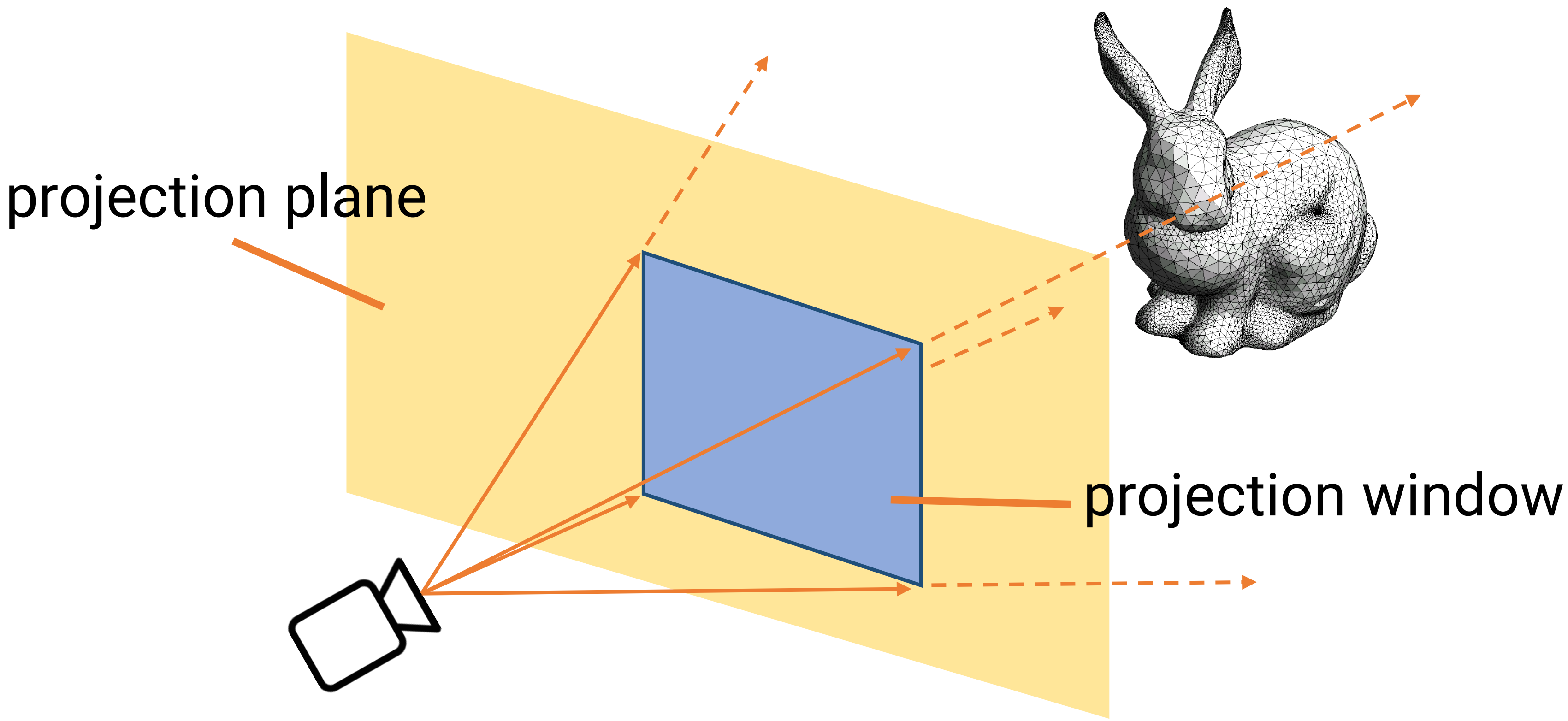


# Perspective Projection (cont.)

- Four components for the perspective projection matrix
  - **The aspect ratio of the screen**
    - The ratio between the width and the height (W/H)
  - **The vertical field of view**
    - The vertical angle of the camera through which we are looking at the world
  - **The location of the near Z plane**
    - Used to clip objects that are too close to the camera
  - **The location of the far Z plane**
    - Used to clip objects that are too distant from the camera

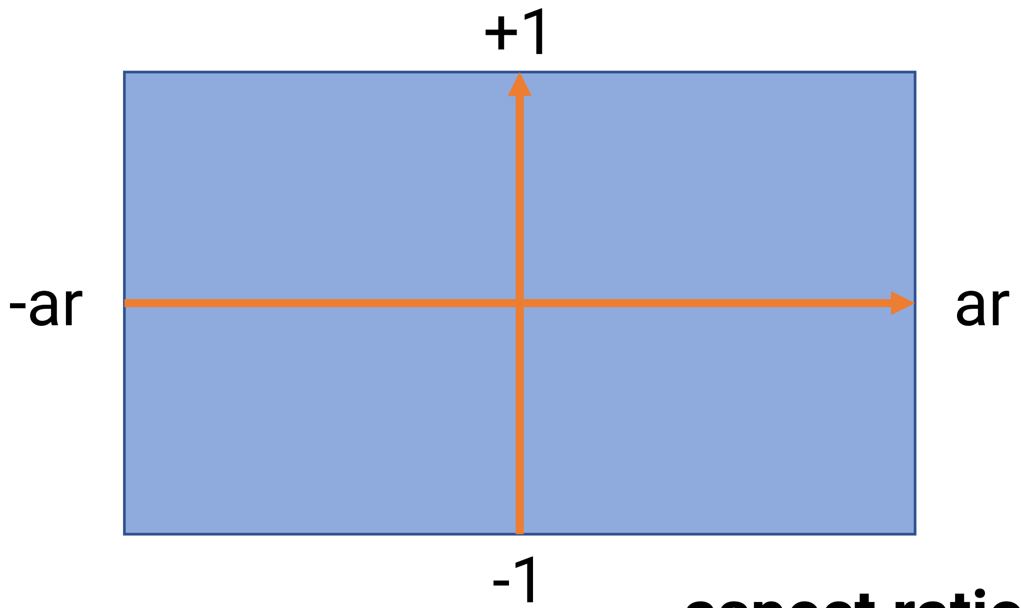
# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - The projection plane and the projection window



# Perspective Projection (cont.)

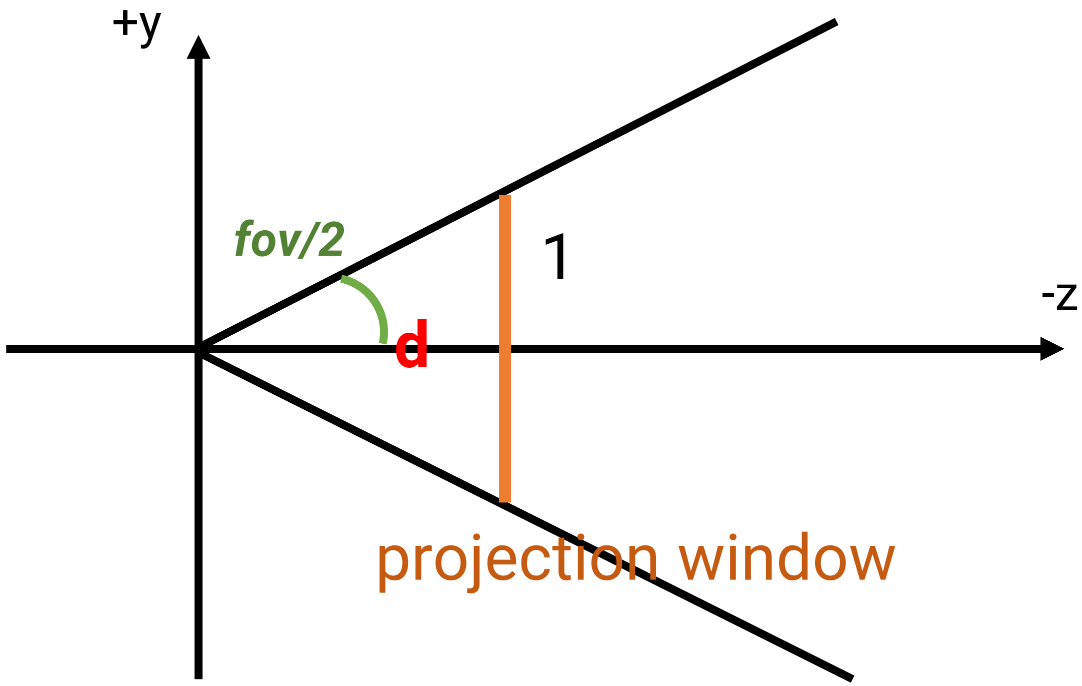
- Derivation of the perspective projection matrix
  - Determine the height of the projection window as 2
  - The width of the projection window becomes 2 times the aspect ratio (ar)




**aspect ratio (ar) = W/H**

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - We can determine the distance from the camera to the projection window based on the field of view (fov)

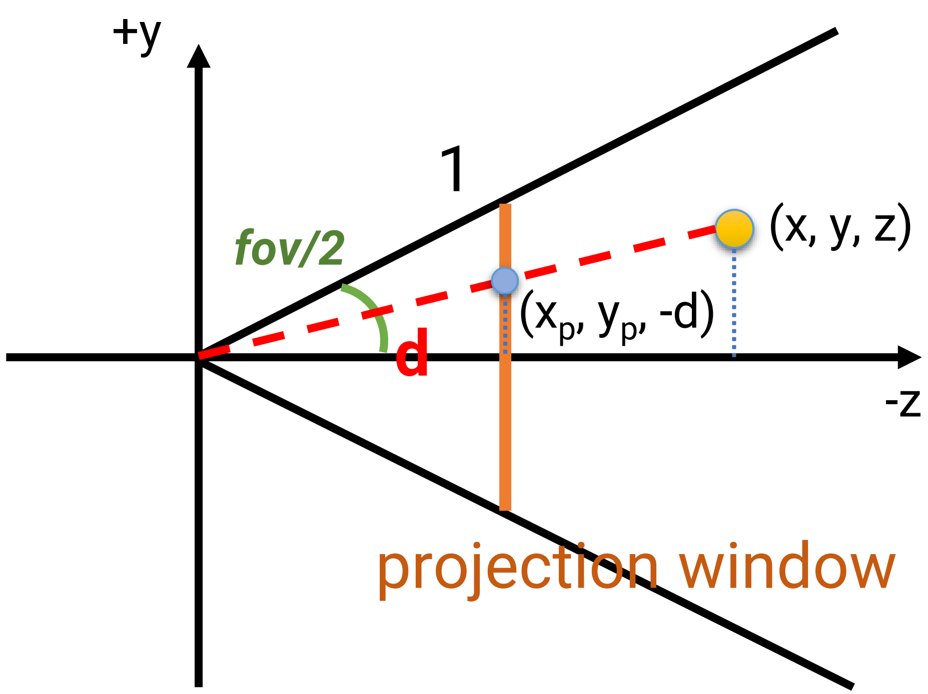



$$\tan(\frac{\alpha}{2}) = \frac{1}{d}$$

$$\Rightarrow d = \frac{1}{\tan(\frac{\alpha}{2})}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Assume we want to find the projected coordinate $(x_p, y_p)$ of a 3D point $(x, y, z)$
  - The y component can be derived as …



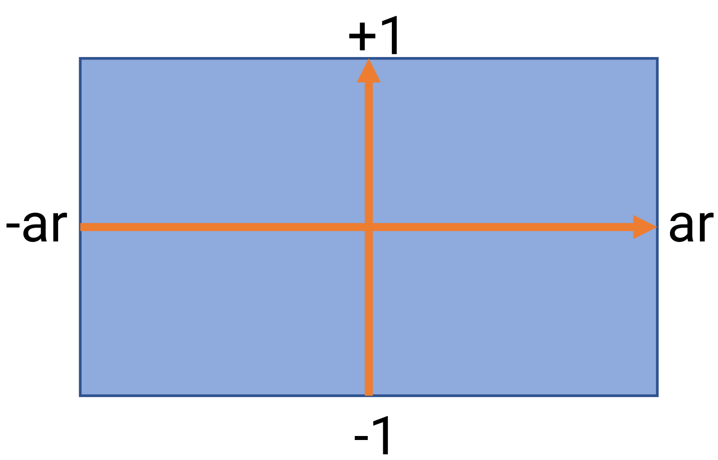

$$\frac{y_p}{d} = \frac{y}{-z}$$

$$\Rightarrow \quad y_p = \frac{y \cdot d}{-z}$$

$$\Rightarrow \quad y_p = \frac{y}{-z \cdot \tan(\frac{\alpha}{2})}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Do the same derivation for the x component
    - Note in the x-direction we have to multiply the aspect ratio ***ar***
  - After that, we can obtain the following equations




$$x_p = \frac{x}{ar \cdot (-z) \cdot \tan(\frac{\alpha}{2})}$$

$$y_p = \frac{y}{-z \cdot \tan(\frac{\alpha}{2})}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Fill-in the matrix, based on the following conditions

$$x_p = \frac{x}{ar \cdot (\boxed{-z}) \cdot \boxed{\tan(\frac{\alpha}{2})}} \qquad y_p = \frac{y}{\boxed{-z} \cdot \boxed{\tan(\frac{\alpha}{2})}}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = \begin{bmatrix} \longleftarrow \mathbf{f(x)} \longrightarrow \\ \longleftarrow \mathbf{f(y)} \longrightarrow \\ \longleftarrow \mathbf{f(z)} \longrightarrow \\ \longleftarrow \mathbf{f(w)} \longrightarrow \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Fill-in the matrix, based on the following conditions

$$x_p = \frac{x}{ar \cdot (\boxed{-z}) \cdot \boxed{\tan(\frac{\alpha}{2})}} \qquad y_p = \frac{y}{\boxed{-z} \cdot \boxed{\tan(\frac{\alpha}{2})}}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{ar \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ \longleftarrow & \mathbf{f(z)} & \longrightarrow & \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Fill-in the matrix, based on the following conditions
    - Assume the Z function has a shape $f(z) = A(-z) + B$
    - After perspective division, it becomes

$$f(z) = A - \frac{B}{z}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{ar \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
  - Fill-in the matrix, based on the following conditions

$$f(-nearZ) = -1 \Rightarrow A - \frac{B}{-nearZ} = -1 \Rightarrow A = -1 - \frac{B}{nearZ}$$

$$f(-farZ) = 1 \Rightarrow A - \frac{B}{-farZ} = 1 \Rightarrow A = 1 - \frac{B}{farZ}$$

---

$$2 = \frac{B}{farZ} - \frac{B}{nearZ}$$

$$\Rightarrow \frac{B \cdot nearZ - B \cdot farZ}{farZ \cdot farZ} = 2$$

$$\Rightarrow B(nearZ - farZ) = 2 \cdot farZ \cdot farZ$$

$$\Rightarrow \boxed{\begin{aligned} B &= \frac{2 \cdot farZ \cdot farZ}{nearZ - farZ} \\ A &= \frac{-nearZ - farZ}{nearZ - farZ} \end{aligned}}$$

# Perspective Projection (cont.)

- Derivation of the perspective projection matrix
    - Fill-in the matrix, based on the following conditions

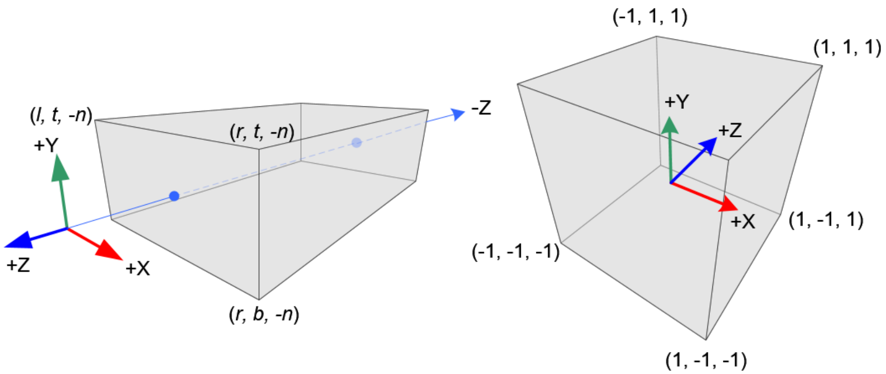
$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{ar \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & \frac{-nearZ - farZ}{nearZ - farZ} & \frac{2 \cdot farZ \cdot nearZ}{nearZ - farZ} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Camera Models Comparison


# Outline

- Introduction to real-world cameras
- Introduction to computer graphics cameras
- Camera space and camera transformation
- Projective cameras
- **OpenGL Implementation**

# Ortho Projection Matrix




$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- glm::mat4x4 ortho( const float left, const float right,
  const float bottom, const float bottom,
  const float near, const float far )

```
glm::mat4x4 goP = glm::ortho(-5.0f, 5.0f, -5.0f, 5.0f, 0.01f, 100.0f);
```

# Perspective Projection Matrix

$$\begin{bmatrix} \frac{1}{ar \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & \frac{-nearZ - farZ}{nearZ - farZ} & \frac{2 \cdot farZ \cdot nearZ}{nearZ - farZ} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

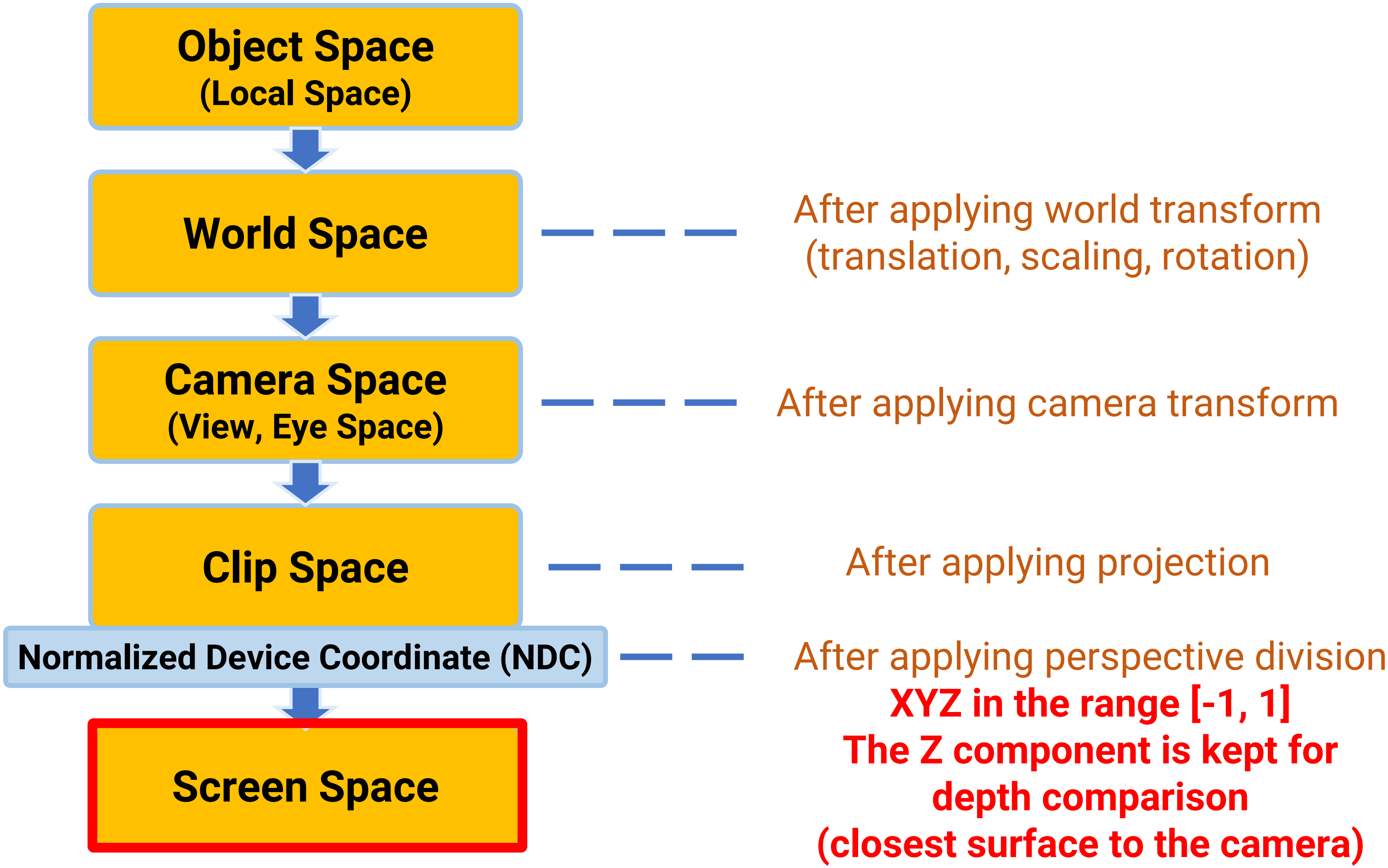
- glm::mat4x4 perspective( const float fovy , const float aspectRatio , const float near , const float far )

use radian, not degree

width / height

```
float fovy = glm::radians(30.0f);
float aspectRatio = 640.0f / 360.0f;
float nearZ = 0.1f;
float farZ = 100.0f;
glm::mat4x4 gP = glm::perspective(fovy, aspectRatio, nearZ, farZ);
```

# The Full Vertex Transform Pipeline



# Apply the Transformation on CPU

- To transform a vertex from object space to clip space, we multiply its position with the **model-view-projection (MVP)** matrix
- We can pre-multiply part of the matrix if some of them are fixed
  - For example, we can pre-multiply the camera (view) and the projection matrix to form a VP matrix, and change the model matrix to perform object animation
- Remember to do the **perspective division**

# Apply the Transformation on CPU (cont.)

```
glm::mat4x4 M = glm::rotate(glm::mat4x4(1.0f), glm::radians(30.0f), glm::vec3(0, 1, 0));

glm::vec3 cameraPos = glm::vec3(0.0f, 0.5f, 2.0f);
glm::vec3 cameraTarget = glm::vec3(0.0f, 0.0f, 0.0f);
glm::vec3 cameraUp = glm::vec3(0.0f, 1.0f, 0.0f);
glm::mat4x4 V = glm::lookAt(cameraPos, cameraTarget, cameraUp);

float fov = 40.0f;
float aspectRatio = (float)screenWidth / (float)screenHeight;
float zNear = 0.1f;
float zFar = 100.0f;
glm::mat4x4 P = glm::perspective(glm::radians(fov), aspectRatio, zNear, zFar);

glm::mat4x4 MVP = P * V * M;

// Apply CPU transformation.
mesh->ApplyTransformCPU(MVP);
```

# Apply the Transformation on CPU (cont.)

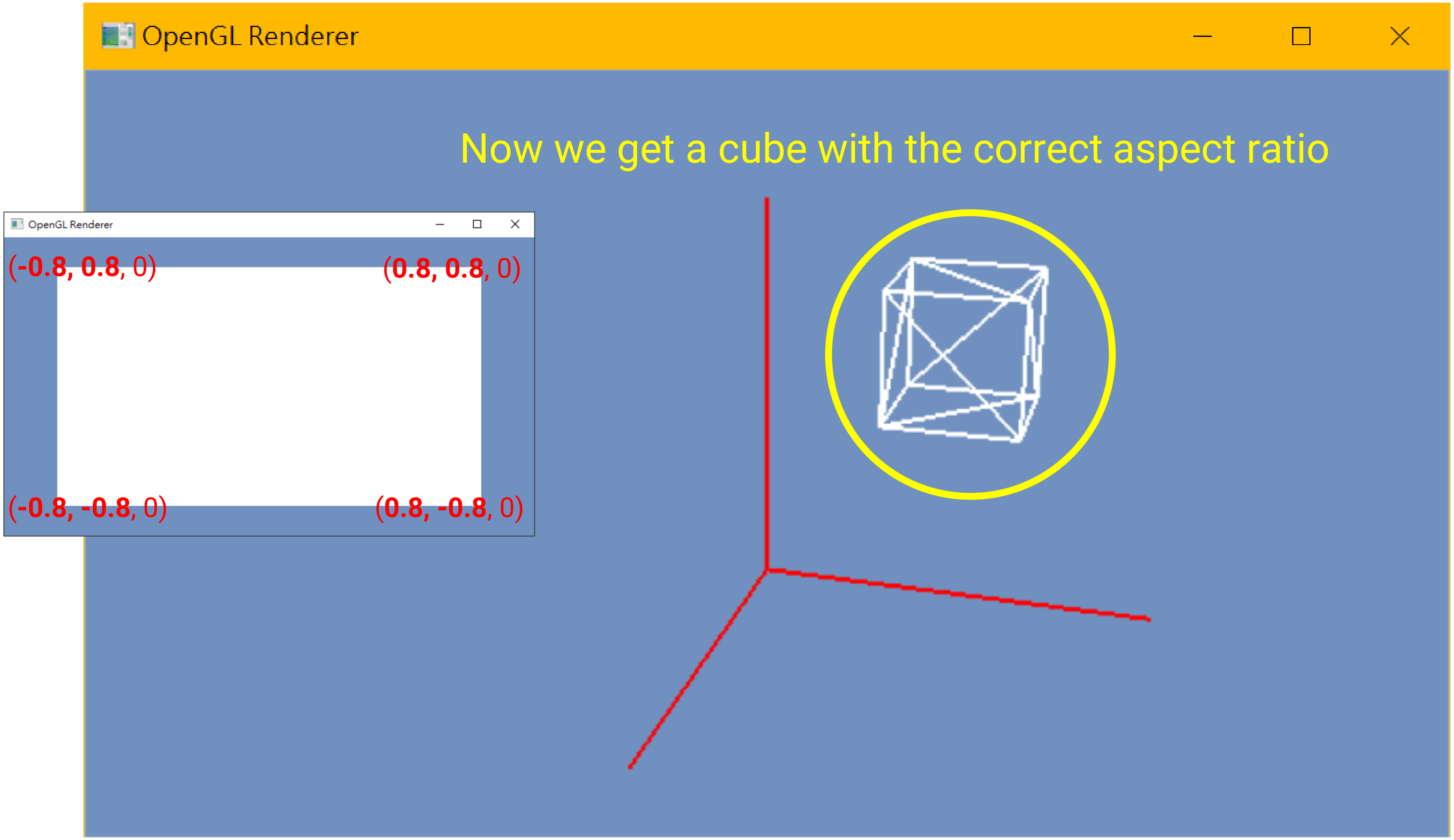
```
void ApplyTransformCPU(std::vector<glm::vec3>& vertexPositions, const glm::mat4x4& mvpMatrix)
{
    for (unsigned int i = 0 ; i < vertexPositions.size(); ++i) {
        glm::vec4 p = mvpMatrix * glm::vec4(vertexPositions[i], 1.0f);
        if (p.w != 0.0f) {
            float inv = 1.0f / p.w;
            vertexPositions[i].x = p.x * inv;
            vertexPositions[i].y = p.y * inv;
            vertexPositions[i].z = p.z * inv;
        }
    }
}
```

perspective division

- A useful coding technique available in shader programming
- It combines a 3d vector and a 1d scalar to form a 4d vector
- You can also write

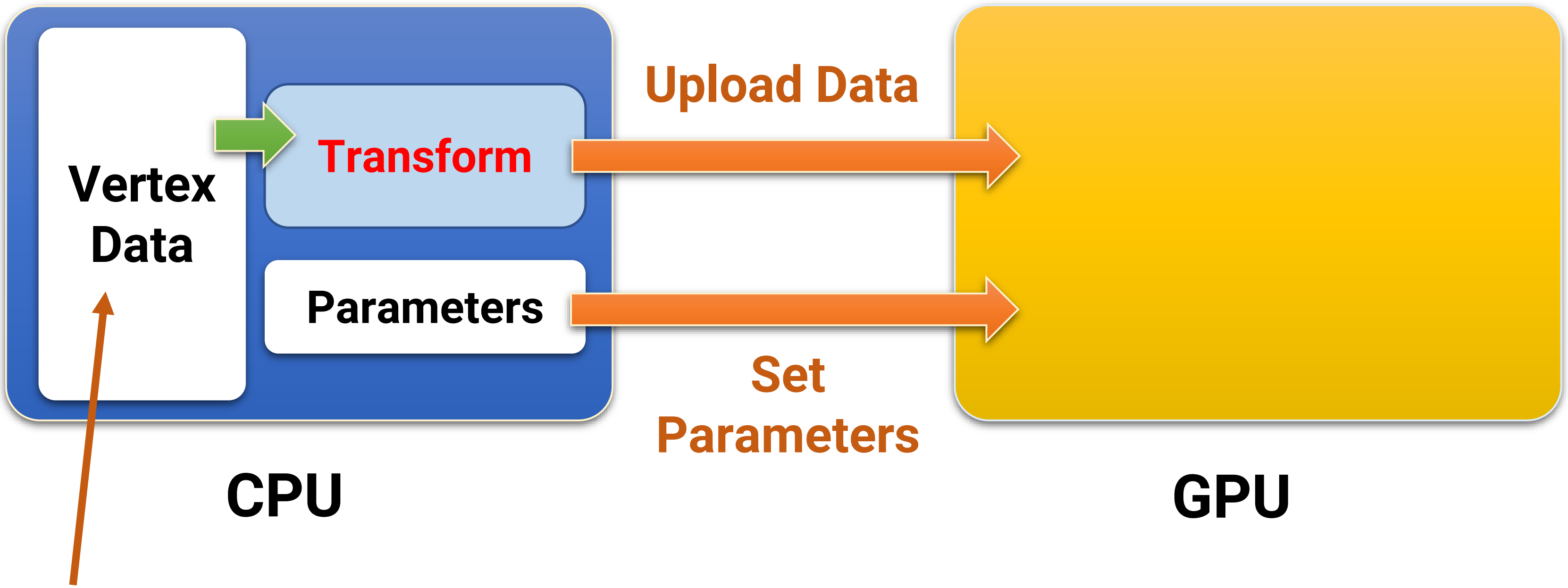
```
glm::vec4(vertexPositions[i].x,
          vertexPositions[i].y,
          vertexPositions[i].z,
          1.0f);
```

# Apply the Transformation on CPU (cont.)



# Apply the Transformation on CPU

- So far, we have performed the transformation of vertices on the CPU

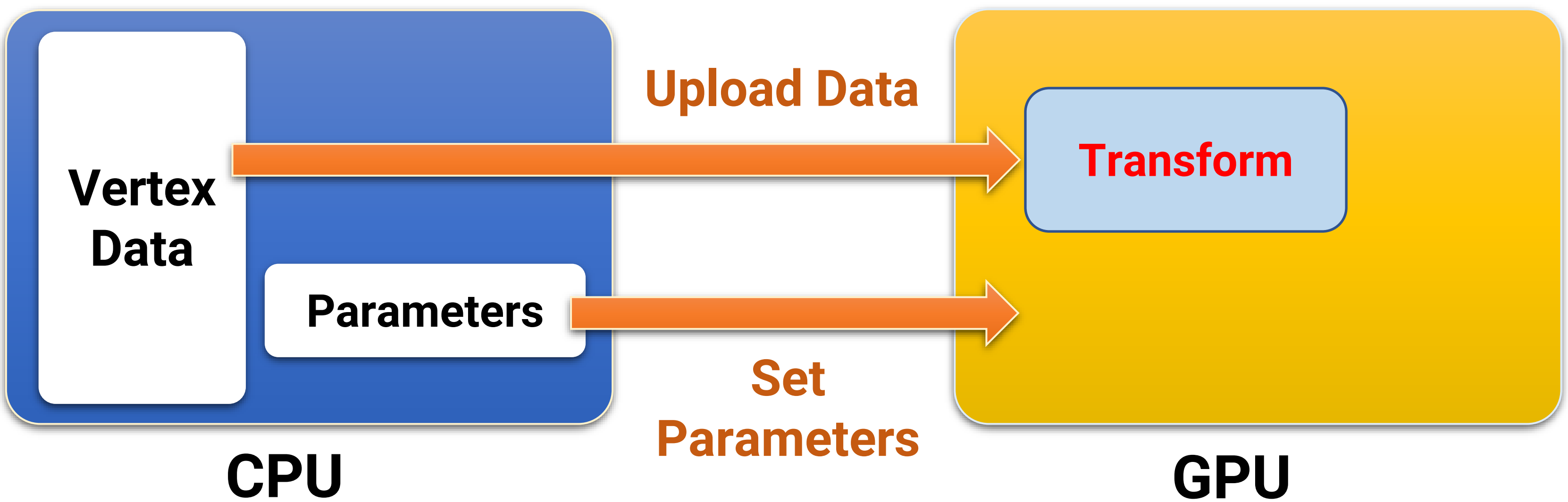



**Vertex attributes**

Position
Normal
Texture coordinate

# Apply the Transformation on GPU

- However, doing this job on CPU is not cost-effective
  - CPU is good at doing sequential, complex jobs
  - But vertex transform is simple and can be done in parallel
- Next class, we will introduce the **GPU graphics pipeline** and the **vertex shaders** for **parallel** processing




END OF PRESENTATION
ANY QUESTIONS?