3D Computer Graphics (II)

Multimedia Techniques & Applications

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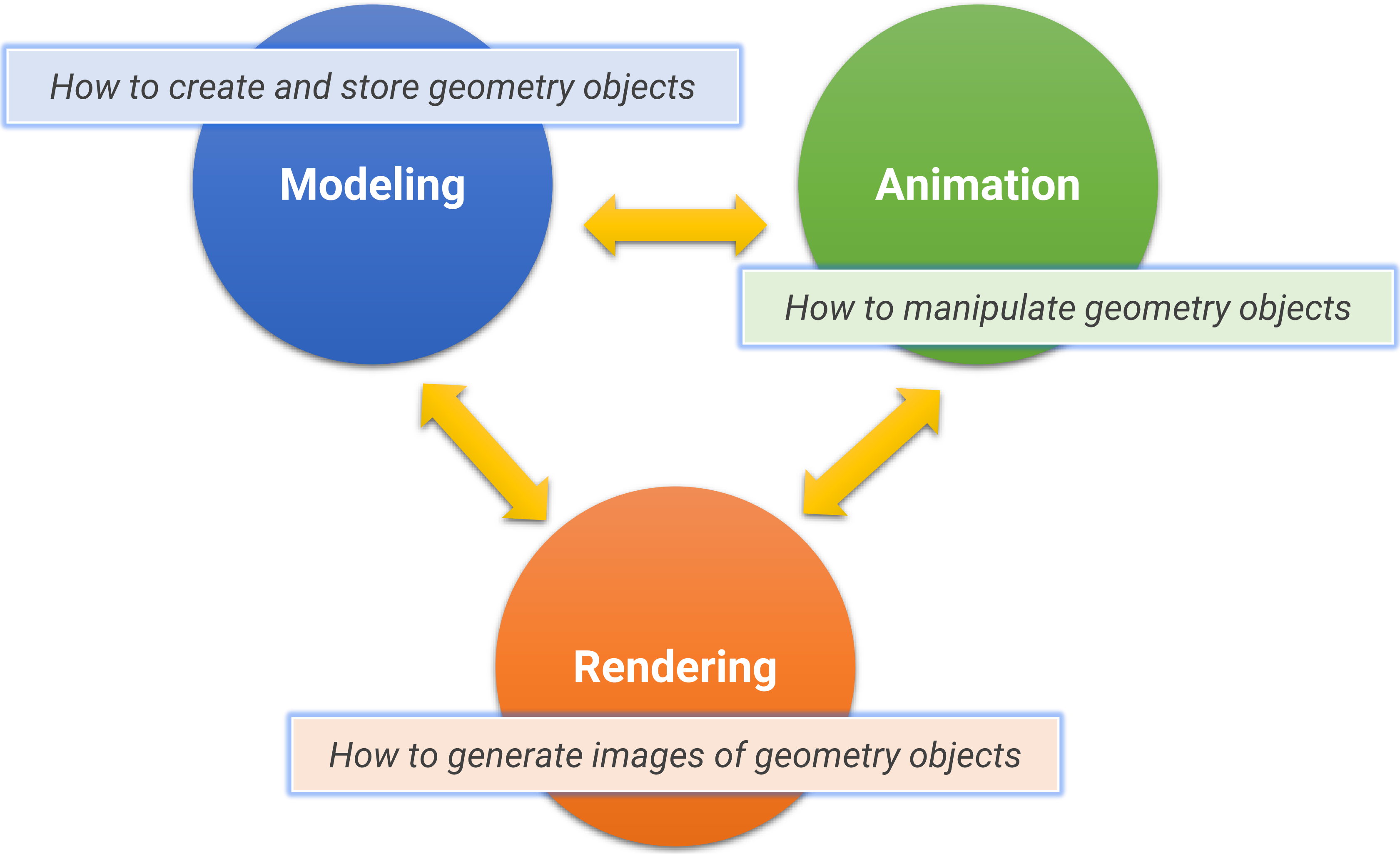
(with slides borrowed from Prof. Yung-Yu Chuang)

What is Computer Graphics

- Computer graphics are pictures and films created using computers
- Computer graphics is the process of creation, storage and manipulation of models and images using data structure and algorithms

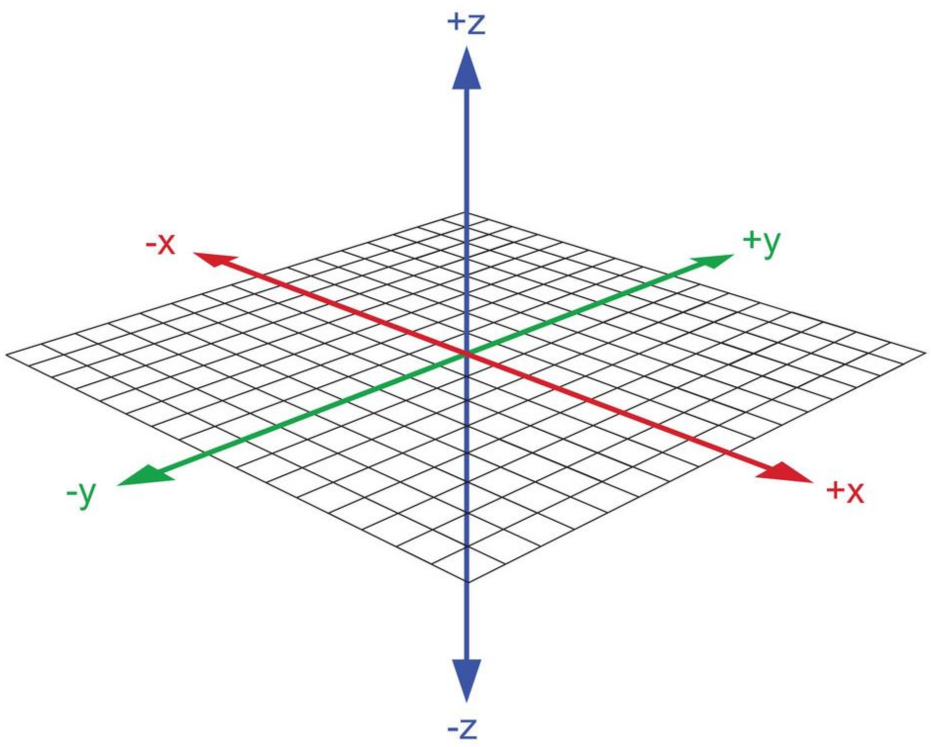
we will focus on this

Major Subfields of Computer Graphics



Description of a 3D World

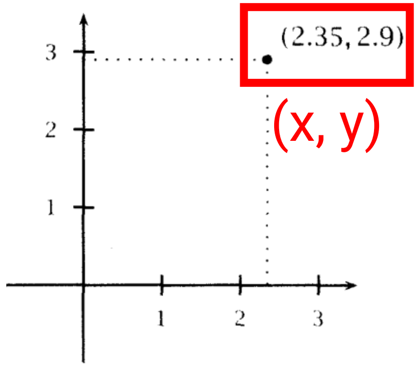
- **3D coordinate system**
  - The formation of x, y, and z axis can be different

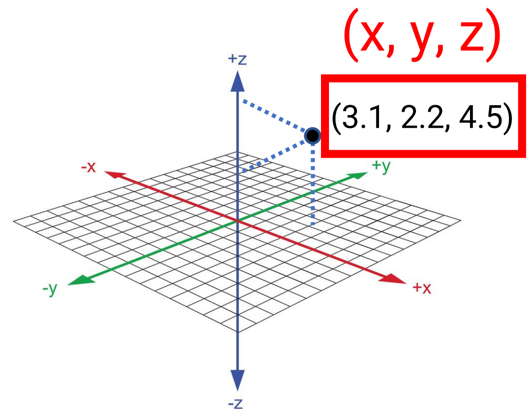


From 2D to 3D

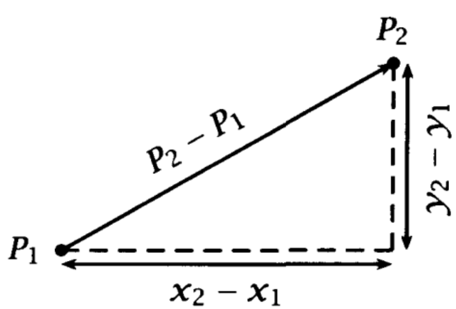
2D | **3D**

point

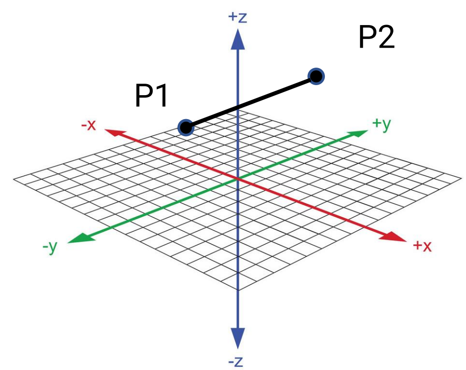




vector



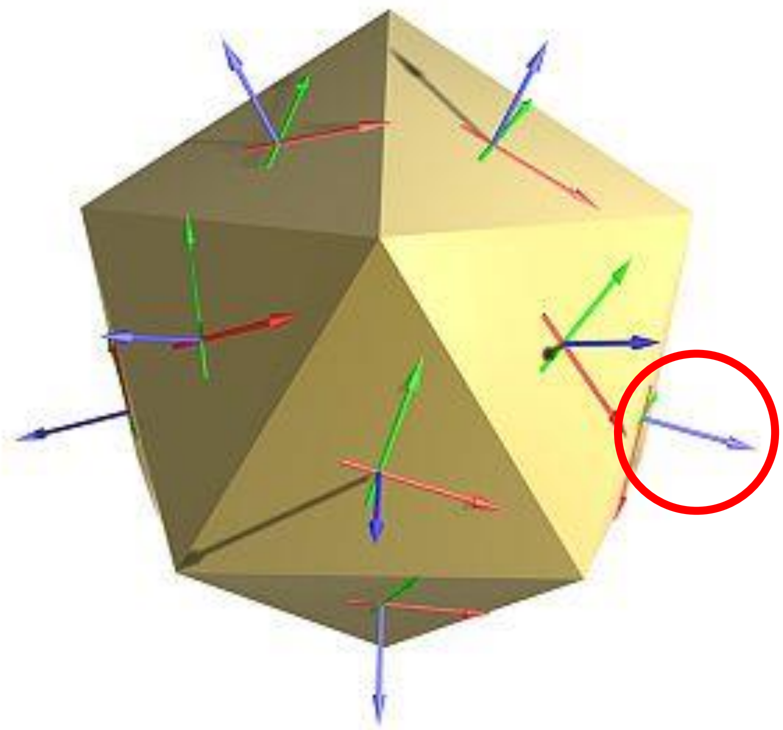
(x2-x1, y2-y1)



(x2-x1, y2-y1, z2-z1)

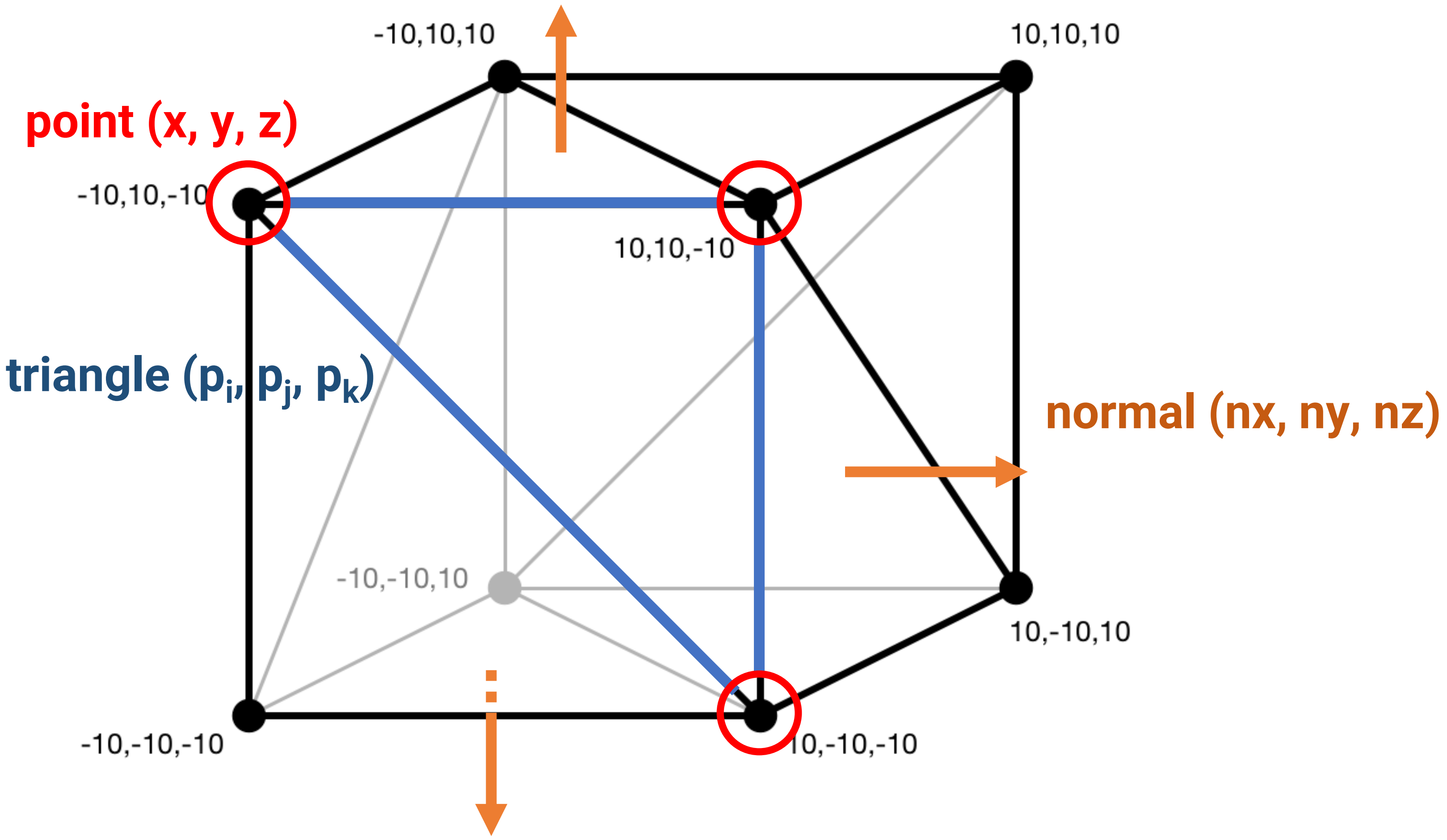
Surface Normal

- A **surface normal** is a vector that is **perpendicular** to a surface at a particular position



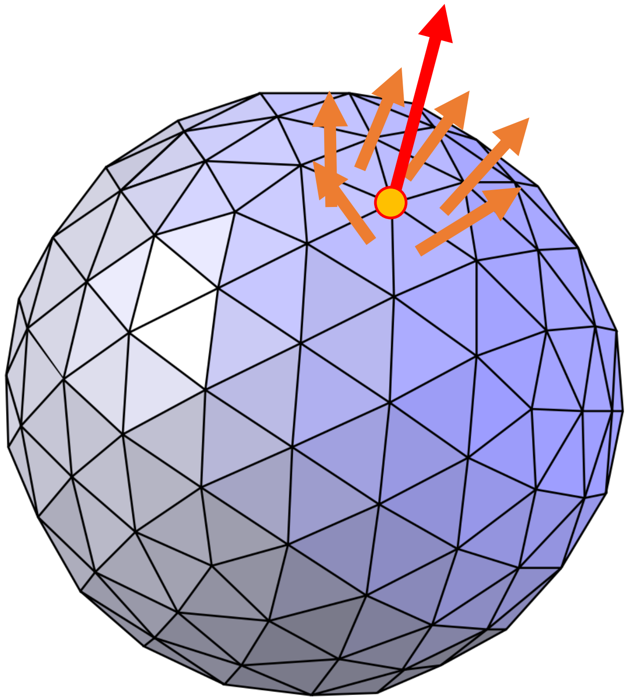
normal (nx, ny, nz)

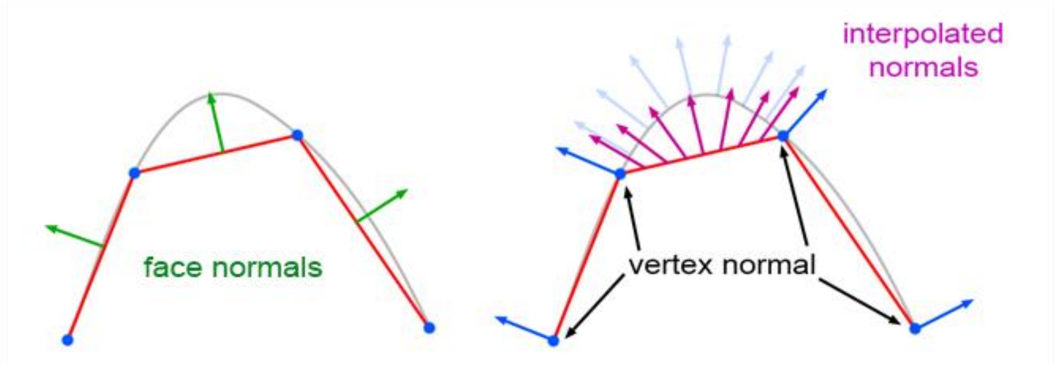
Points, Normals, and Triangles

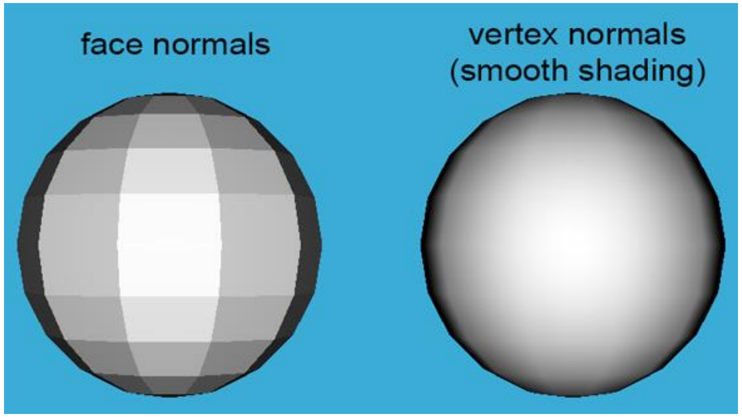


Vertex Normal

- Sometimes we also define the normal of a vertex by averaging the surface normals of its adjacent faces for **smooth shading**

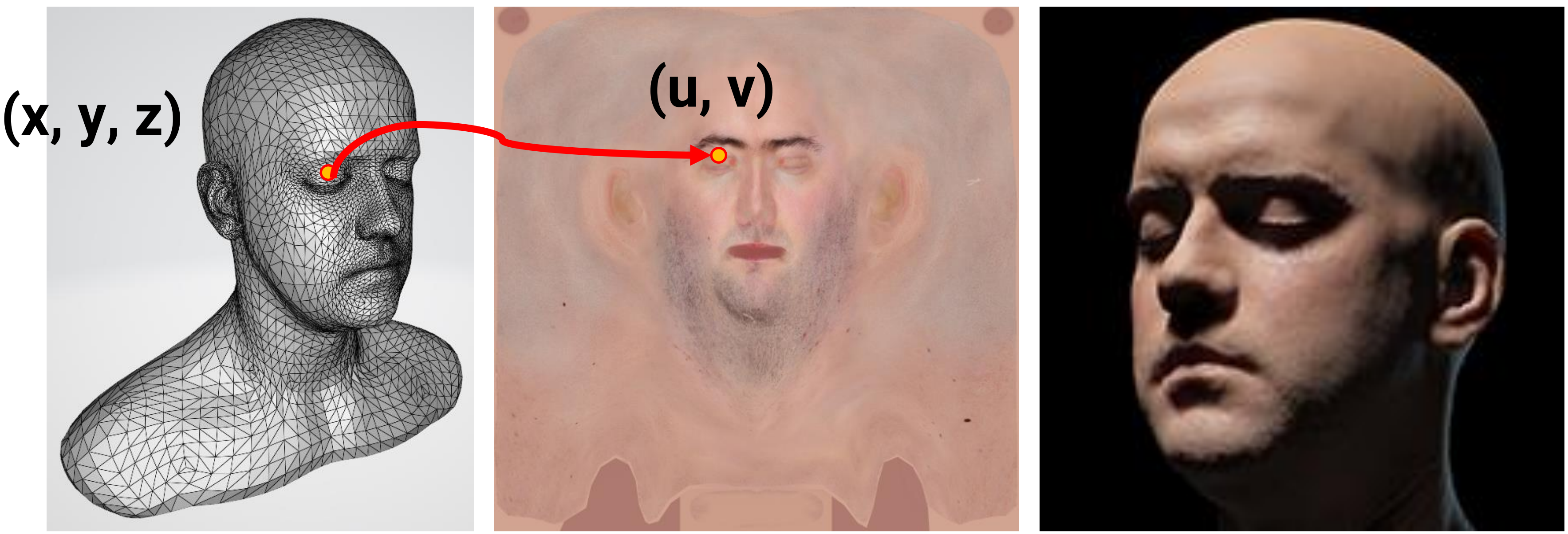






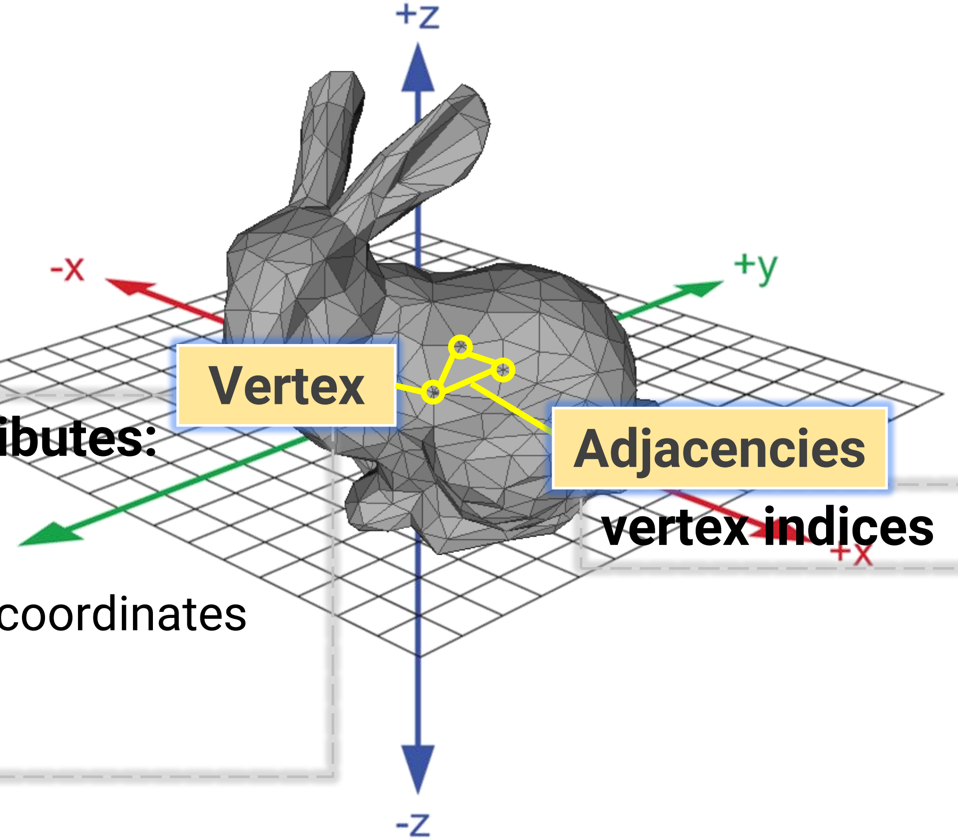
Texture Coordinate

- **Texture**
  - Used to represent spatially-varying data
  - Decouple materials from geometry
  - Need a mapping from 3D (xyz, object space) to 2D (uv, image space), called **texture coordinate**



Description of an 3D Object

- We will focus on triangle mesh today
- Define the **position**, **normal**, and **texture coordinate** on the **vertices of triangles**



vertex attributes:

- Position
- Normal
- Texture coordinates
- …

vertex indices

Common 3D Model Format

- **Wavefront (*.obj)**
- Polygon file format (*.ply)
- **Filmbox (*.fbx)**
- MAX (*.max)
- Digital Asset Exchange File (*.dae)
- STereoLithography (*.stl)

Wavefront (OBJ)

```
mtllib default.mtl
```
material file

```
v 0.5 0.5 -0.5
v 0.5 0.5 0.5
v -0.5 0.5 0.5
v -0.5 0.5 -0.5
v 0.5 -0.5 -0.5
v 0.5 -0.5 0.5
v -0.5 -0.5 0.5
v -0.5 -0.5 -0.5
```
vertex position

```
vt 0 1
vt 0 0
vt 1 0
vt 1 1
```
vertex texture coordinate

```
vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0
```
vertex normal

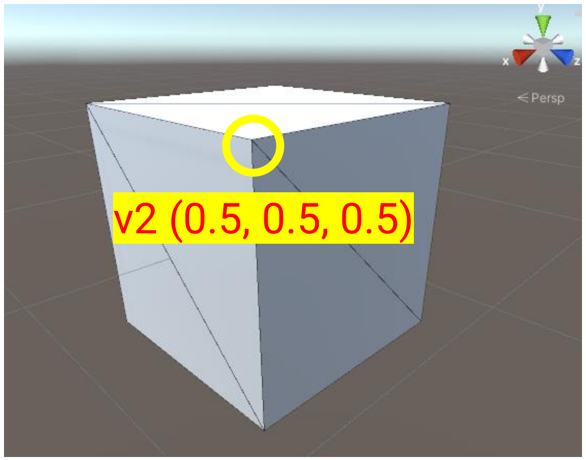
```
g cube
```
group name

```
usemtl default
```
group material

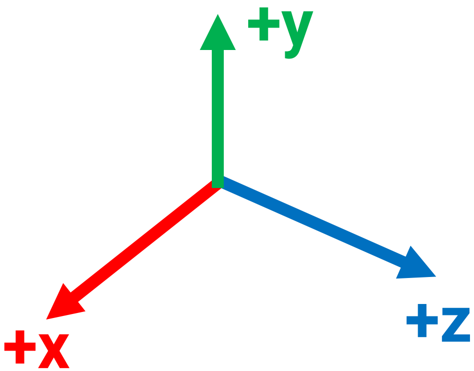
```
f  8/4/6  7/3/6  6/2/6
f  8/4/6  6/2/6  5/1/6
f  8/4/5  4/3/5  3/2/5
f  8/4/5  3/2/5  7/1/5
f  6/4/4  2/3/4  1/2/4
f  6/4/4  1/2/4  5/1/4
f  5/4/3  1/3/3  4/2/3
f  5/4/3  4/2/3  8/1/3
f  7/4/2  3/3/2  2/2/2
f  7/4/2  2/2/2  6/1/2
f  3/4/1  4/3/1  1/2/1
f  3/4/1  1/2/1  2/1/1
```
face adjacencies

cube.obj

The Simplest OBJ File



a unit cube



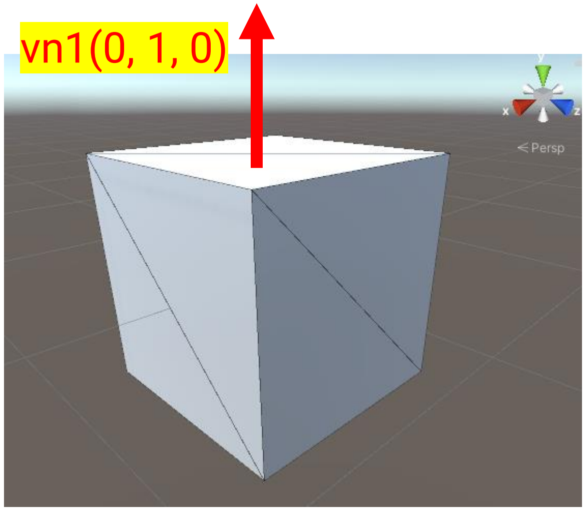
vertex position x y z

```
1  v 0.5 0.5 -0.5
2  v 0.5 0.5 0.5
3  v -0.5 0.5 0.5
4  v -0.5 0.5 -0.5
5  v 0.5 -0.5 -0.5
6  v 0.5 -0.5 0.5
7  v -0.5 -0.5 0.5
8  v -0.5 -0.5 -0.5
```

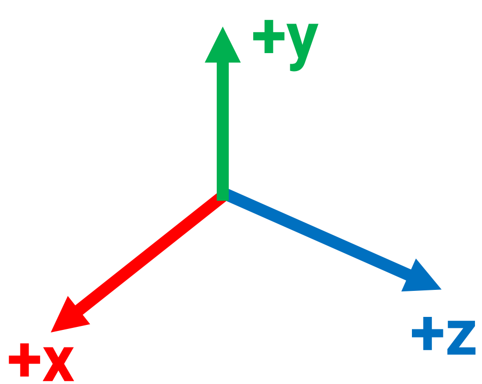
```
mtllib default.mtl

v 0.5 0.5 -0.5
v 0.5 0.5 0.5
v -0.5 0.5 0.5
v -0.5 0.5 -0.5
v 0.5 -0.5 -0.5
v 0.5 -0.5 0.5
v -0.5 -0.5 0.5
v -0.5 -0.5 -0.5

vt 0 1
vt 0 0
vt 1 0
vt 1 1

vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0

g cube
usemtl default

f  8/4/6  7/3/6  6/2/6
f  8/4/6  6/2/6  5/1/6
f  8/4/5  4/3/5  3/2/5
f  8/4/5  3/2/5  7/1/5
f  6/4/4  2/3/4  1/2/4
f  6/4/4  1/2/4  5/1/4
f  5/4/3  1/3/3  4/2/3
f  5/4/3  4/2/3  8/1/3
f  7/4/2  3/3/2  2/2/2
f  7/4/2  2/2/2  6/1/2
f  3/4/1  4/3/1  1/2/1
f  3/4/1  1/2/1  2/1/1
```

cube.obj

The Simplest OBJ File (cont.)



a unit cube



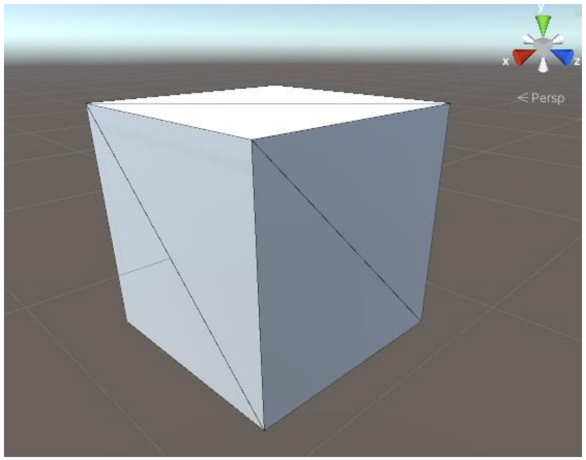
vertex normal nx ny nz

```
1  vn 0 1 0
2  vn -1 0 0
3  vn 1 0 0
4  vn 0 0 -1
5  vn 0 0 1
6  vn 0 -1 0
```

```
mtllib default.mtl

v 0.5 0.5 -0.5
v 0.5 0.5 0.5
v -0.5 0.5 0.5
v -0.5 0.5 -0.5
v 0.5 -0.5 -0.5
v 0.5 -0.5 0.5
v -0.5 -0.5 0.5
v -0.5 -0.5 -0.5

vt 0 1
vt 0 0
vt 1 0
vt 1 1

vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0

g cube
usemtl default

f  8/4/6  7/3/6  6/2/6
f  8/4/6  6/2/6  5/1/6
f  8/4/5  4/3/5  3/2/5
f  8/4/5  3/2/5  7/1/5
f  6/4/4  2/3/4  1/2/4
f  6/4/4  1/2/4  5/1/4
f  5/4/3  1/3/3  4/2/3
f  5/4/3  4/2/3  8/1/3
f  7/4/2  3/3/2  2/2/2
f  7/4/2  2/2/2  6/1/2
f  3/4/1  4/3/1  1/2/1
f  3/4/1  1/2/1  2/1/1
```

cube.obj

The Simplest OBJ File (cont.)



a unit cube

vt2(0, 0) vt3(1, 0)

vt1(0, 1) texture vt4(1, 1)

vertex texture coordinate

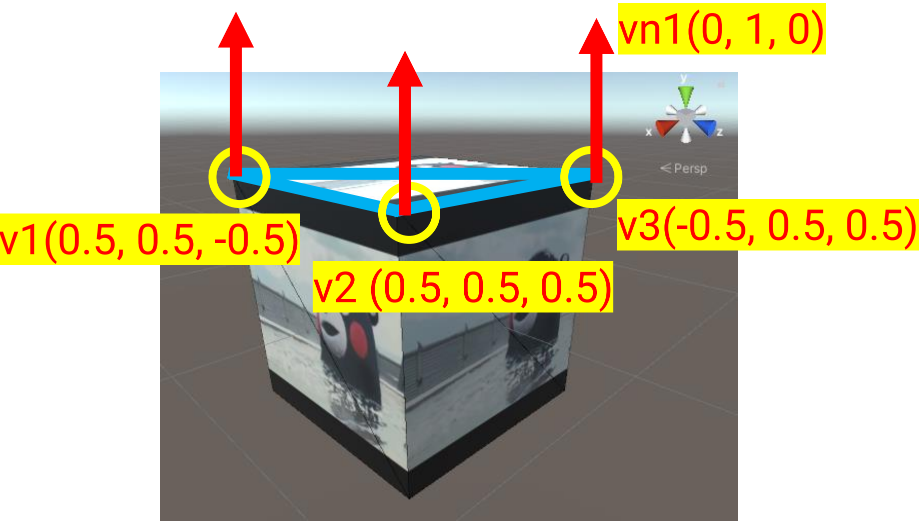
u v

```
1  vt 0 1
2  vt 0 0
3  vt 1 0
4  vt 1 1
```

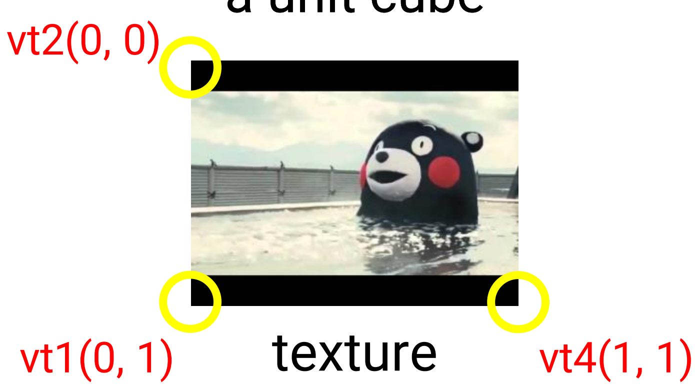
```
mtllib default.mtl

v 0.5 0.5 -0.5
v 0.5 0.5 0.5
v -0.5 0.5 0.5
v -0.5 0.5 -0.5
v 0.5 -0.5 -0.5
v 0.5 -0.5 0.5
v -0.5 -0.5 0.5
v -0.5 -0.5 -0.5

vt 0 1
vt 0 0
vt 1 0
vt 1 1

vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0

g cube
usemtl default

f  8/4/6  7/3/6  6/2/6
f  8/4/6  6/2/6  5/1/6
f  8/4/5  4/3/5  3/2/5
f  8/4/5  3/2/5  7/1/5
f  6/4/4  2/3/4  1/2/4
f  6/4/4  1/2/4  5/1/4
f  5/4/3  1/3/3  4/2/3
f  5/4/3  4/2/3  8/1/3
f  7/4/2  3/3/2  2/2/2
f  7/4/2  2/2/2  6/1/2
f  3/4/1  4/3/1  1/2/1
f  3/4/1  1/2/1  2/1/1
```

cube.obj

The Simplest OBJ File (cont.)



a unit cube



texture

face adjacency
v1/vt1/vn1
(index)

```
f   8/4/6   7/3/6   6/2/6
f   8/4/6   6/2/6   5/1/6
f   8/4/5   4/3/5   3/2/5
f   8/4/5   3/2/5   7/1/5
f   6/4/4   2/3/4   1/2/4
f   6/4/4   1/2/4   5/1/4
f   5/4/3   1/3/3   4/2/3
f   5/4/3   4/2/3   8/1/3
f   7/4/2   3/3/2   2/2/2
f   7/4/2   2/2/2   6/1/2
f   3/4/1   4/3/1   1/2/1
f   3/4/1   1/2/1   2/1/1
```

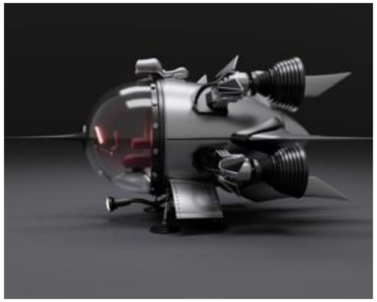
```
mtllib default.mtl

v 0.5 0.5 -0.5
v 0.5 0.5 0.5
v -0.5 0.5 0.5
v -0.5 0.5 -0.5
v 0.5 -0.5 -0.5
v 0.5 -0.5 0.5
v -0.5 -0.5 0.5
v -0.5 -0.5 -0.5

vt 0 1
vt 0 0
vt 1 0
vt 1 1

vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0

g cube
usemtl default
```


```
f  8/4/6  7/3/6  6/2/6
f  8/4/6  6/2/6  5/1/6
f  8/4/5  4/3/5  3/2/5
f  8/4/5  3/2/5  7/1/5
f  6/4/4  2/3/4  1/2/4
f  6/4/4  1/2/4  5/1/4
f  5/4/3  1/3/3  4/2/3
f  5/4/3  4/2/3  8/1/3
f  7/4/2  3/3/2  2/2/2
f  7/4/2  2/2/2  6/1/2
f  3/4/1  4/3/1  1/2/1
f  3/4/1  1/2/1  2/1/1
```


cube.obj

Play with Mesh Viewer

- There are lots of free models on the internet

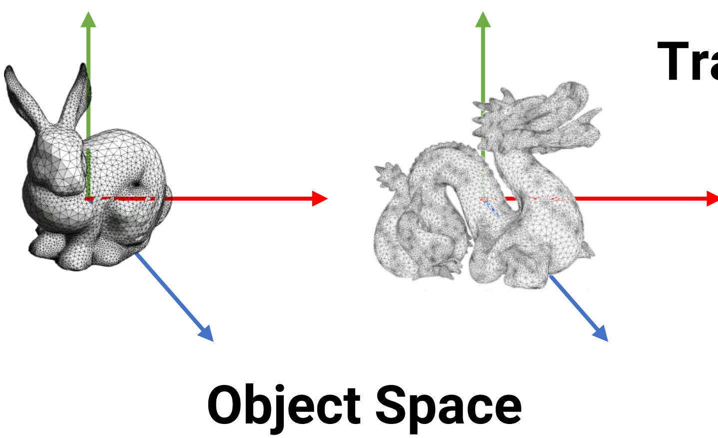



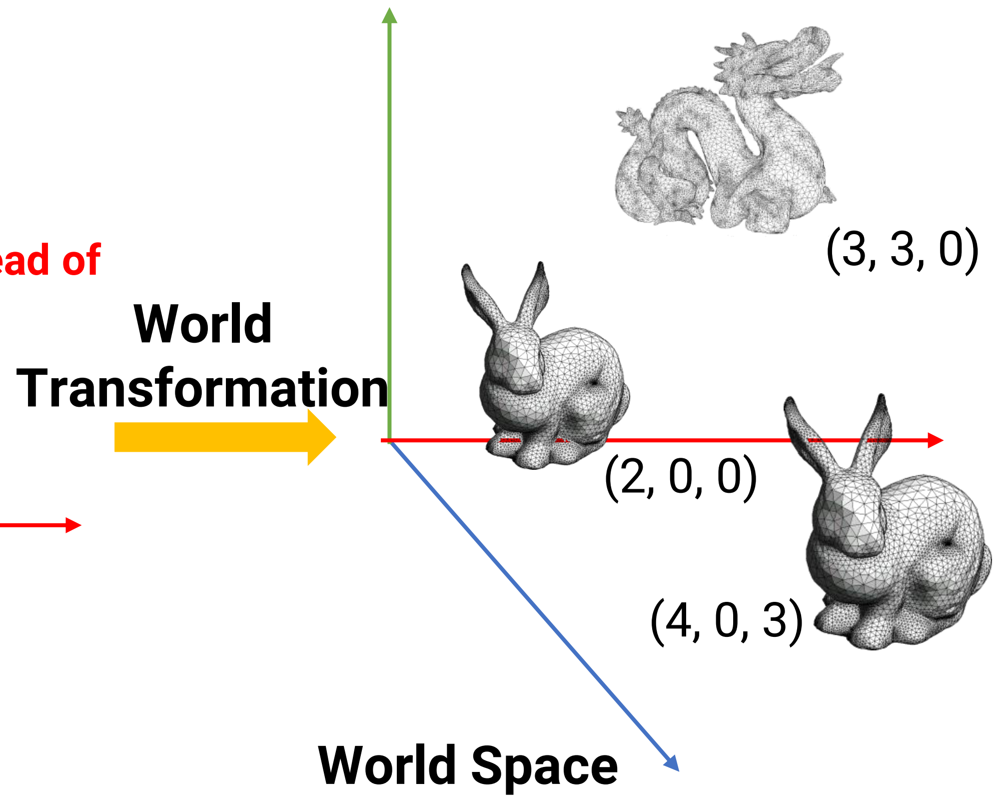
- Try to download one and play with it in the viewer
- Free mesh viewer
  - Windows 3D Viewer
  - Meshlab https://www.meshlab.net/
  - Blender (modeling tool) https://www.blender.org/
  - Unity (game engine)
  - Unreal engine (game engine)

Object Space and World Space

- Shapes (or objects) are defined in **object space** and transformed to **world space**
- Why?
  - Reuse model
  - Object instancing

Store a 4x4 matrix instead of an entire model





Object Space and World Space (cont.)

- Demo with Unity

Recap: 2D Transformation

- Using **3x3 matrix** allows us to perform all transformations using matrix/vector multiplications
  - We can also **pre-multiply** all the matrices together
- We call the (x, y, 1) representation for (x, y) **homogeneous coordinate**

Translation
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

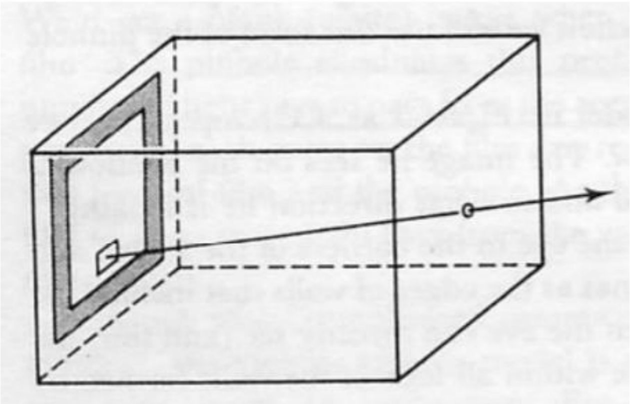
3D Transformation

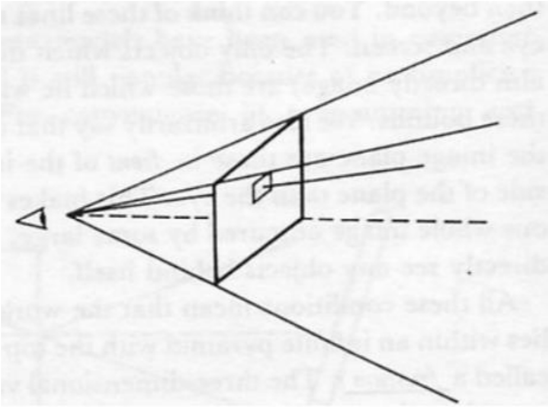
- A 3D transformation ***T*** is represented as a **4x4 matrix**, with **homogeneous coordinate**

	2D	3D
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Scaling	$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & t_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} cos(v) & -sin(v) & 0 & 0 \\ sin(v) & cos(v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(Virtual) Camera

- For most cases in computer graphics, we use a **perspective pinhole** **camera model** for its simplicity

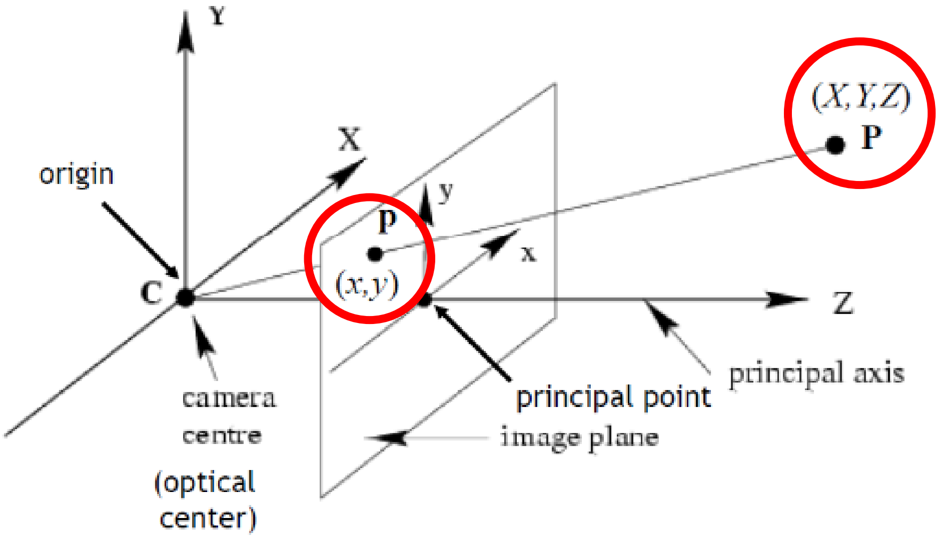


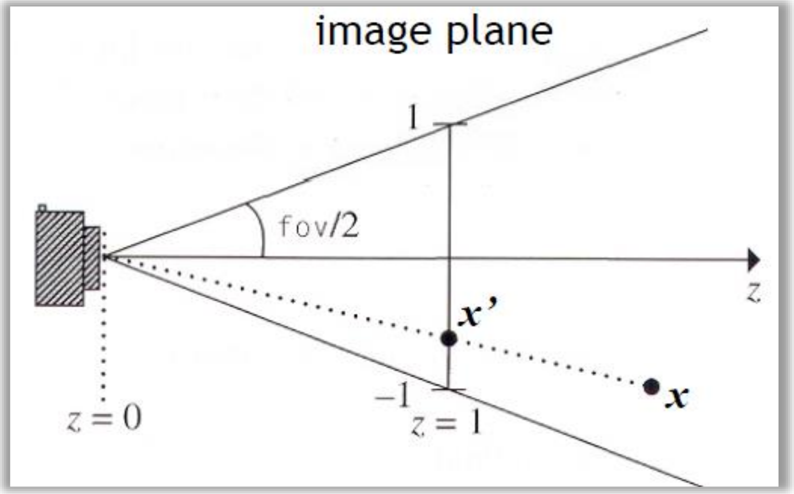


The virtual film is placed **in front of** the camera for avoiding **up-side-down** image

Perspective Pinhole Camera Model

- Assume the camera is located at **origin** and look to **+Z** (or another axis depends on the graphics system)

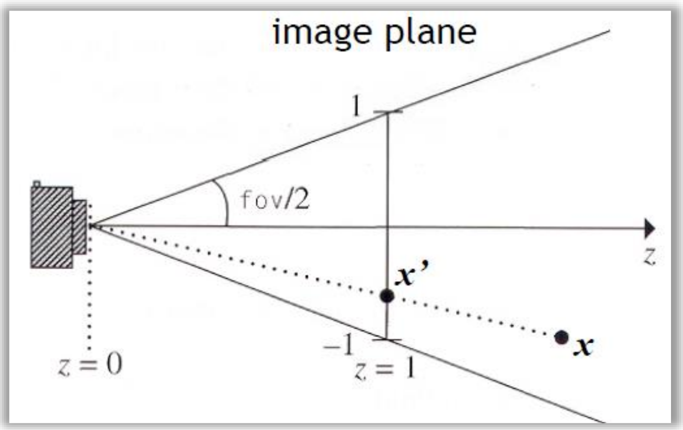


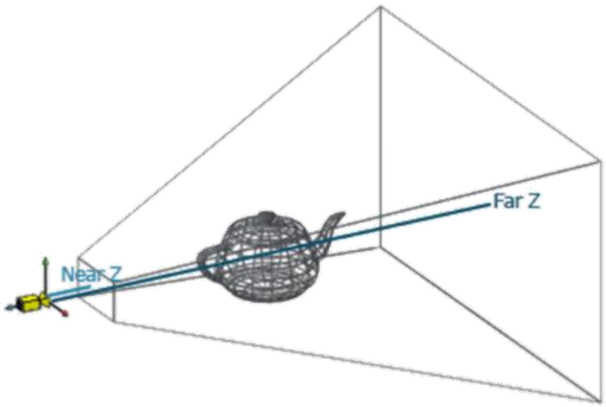


Perspective Pinhole Camera Model (cont.)

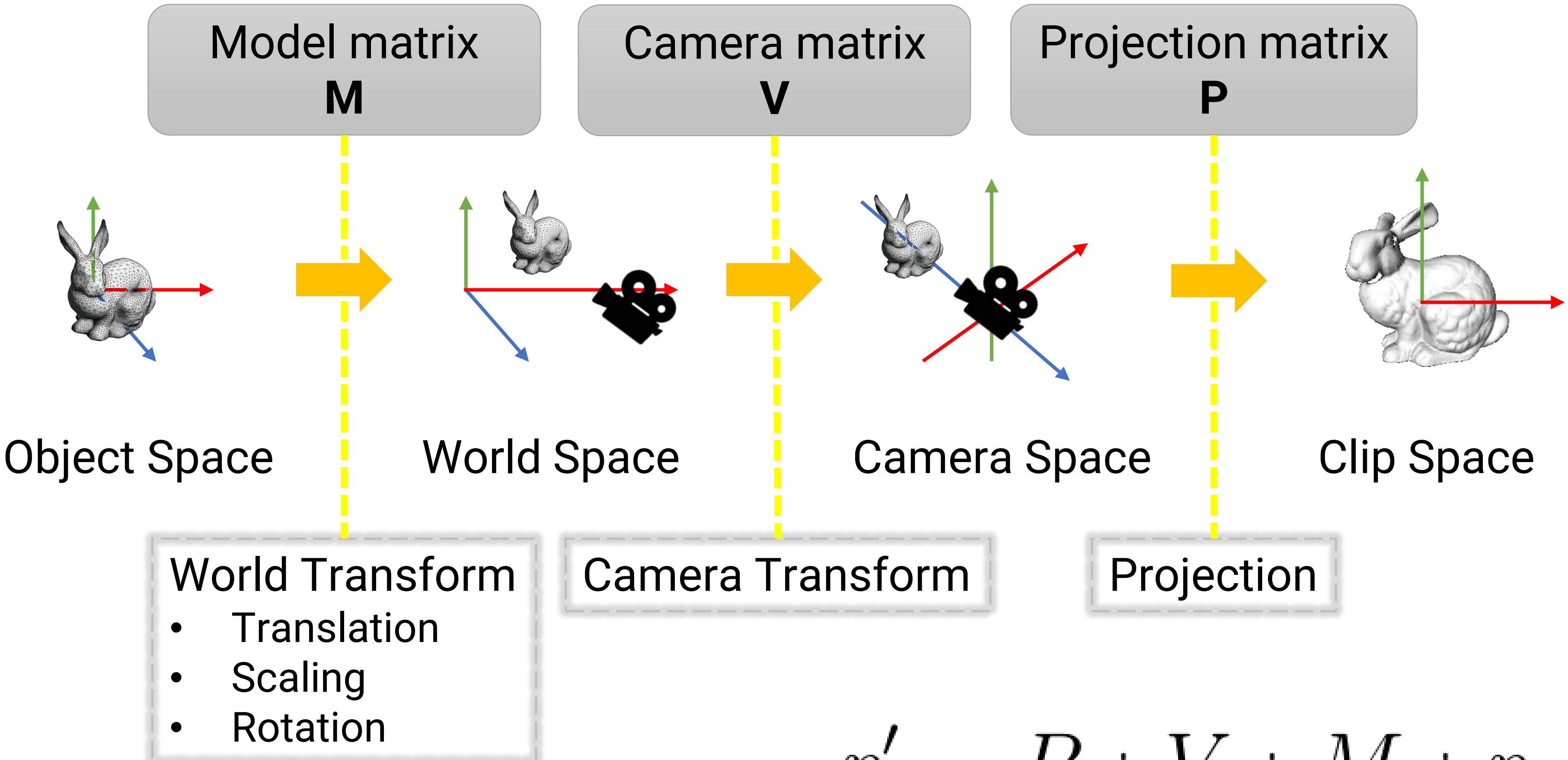
- 3D points can be projected on the virtual film by using a projection matrix

$$\begin{bmatrix} \tan^{-1}(\frac{\mathrm{FOV}_x}{2}) & 0 & 0 & 0 \\ 0 & \tan^{-1}(\frac{\mathrm{FOV}_y}{2}) & 0 & 0 \\ 0 & 0 & \frac{Z_{\mathrm{far}}}{Z_{\mathrm{far}} - Z_{\mathrm{near}}} & -\frac{Z_{\mathrm{far}} Z_{\mathrm{near}}}{Z_{\mathrm{far}} - Z_{\mathrm{near}}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$





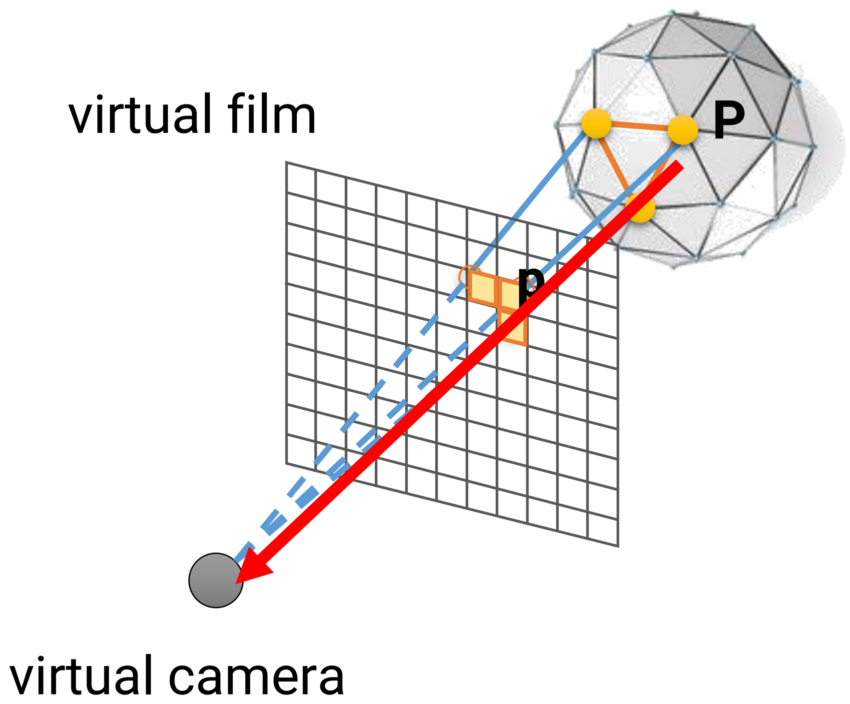
Transform between Spaces

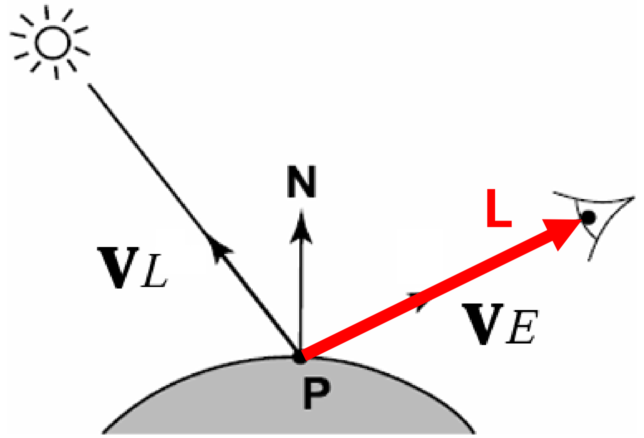


$$p' = P * V * M * p$$

Shading

- Simulate the **interaction** between a **light** and a **surface point**





Point **P** on a surface through a pixel **p**

Normal **N** at **P**

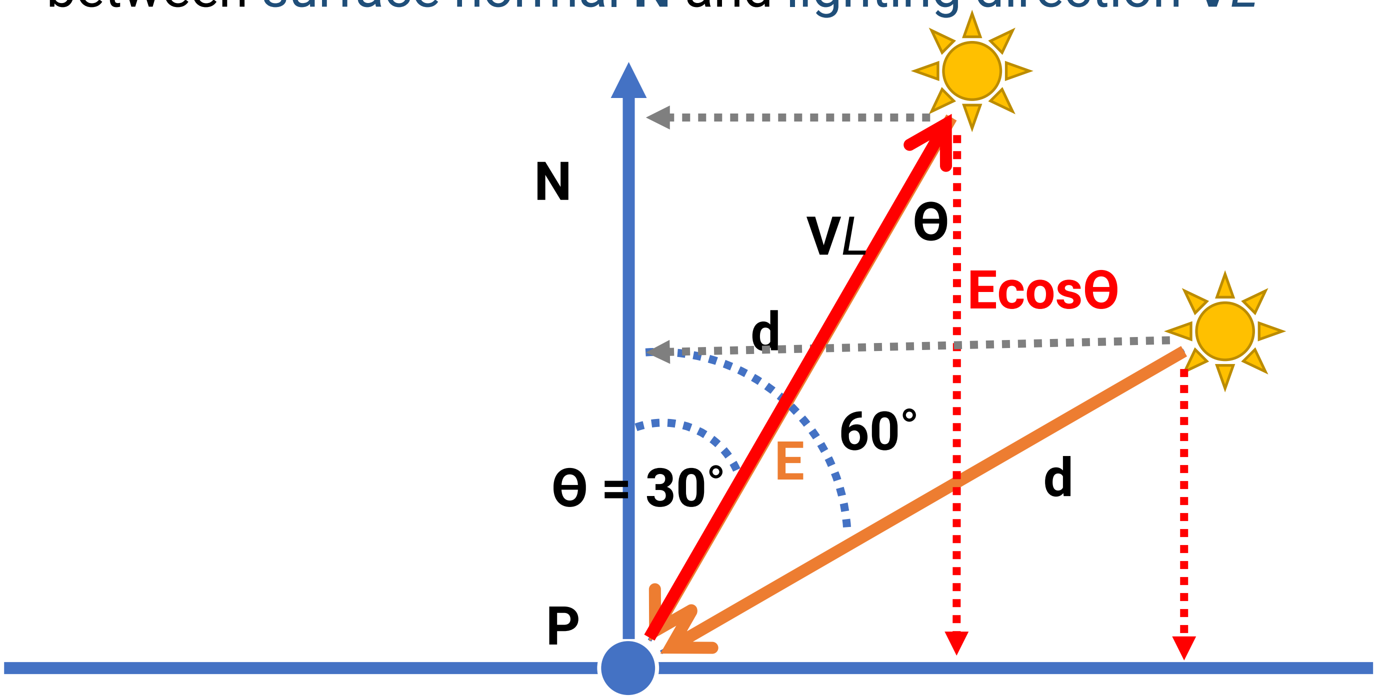
Lighting direction **V***L*

Viewing direction **V***E*

Goal: compute color **L** for pixel **p**

Lambertian Term

- Assume the two lights have equal intensity and equal distance to **P**, which light can contribute more to the point **P**?
  - The contribution is proportional to **cosθ**, where **θ** is the angle between surface normal **N** and lighting direction **V*L***



Virtual Lights in Computer Graphics

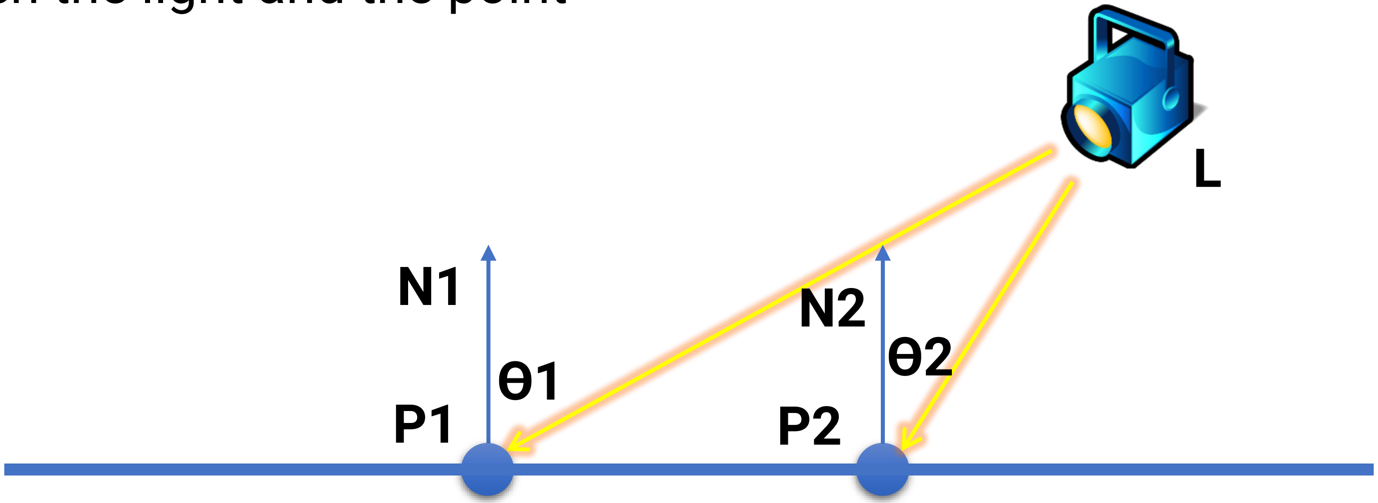
- Point light
- Spot light
- Area light
- Directional light
- Environment light

local lights (Point light, Spot light, Area light)

distant lights (Directional light, Environment light)

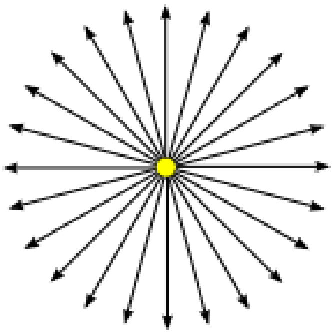
Local Lights

- The distance between a light and a surface is **not** long enough compared to the scene scale
- The position of a light need to be taken into account during shading
  - **Lighting direction = |L – P|**
  - **Lighting attenuation** is proportional to the square of distance between the light and the point

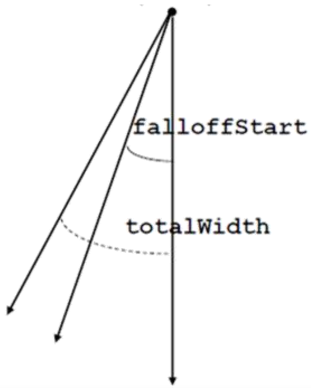


Local Lights (cont.)

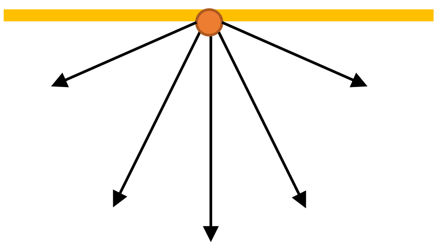
Point Light

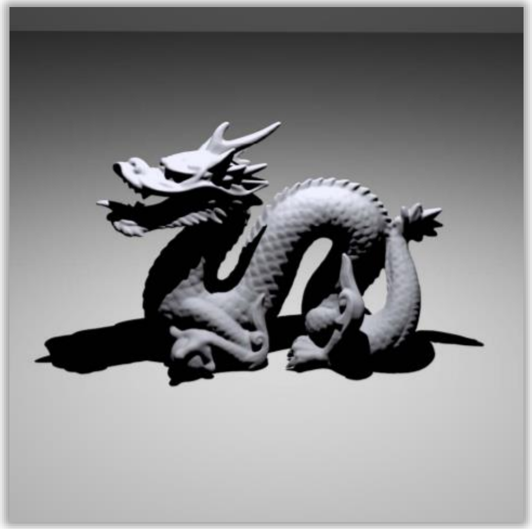


Spot Light



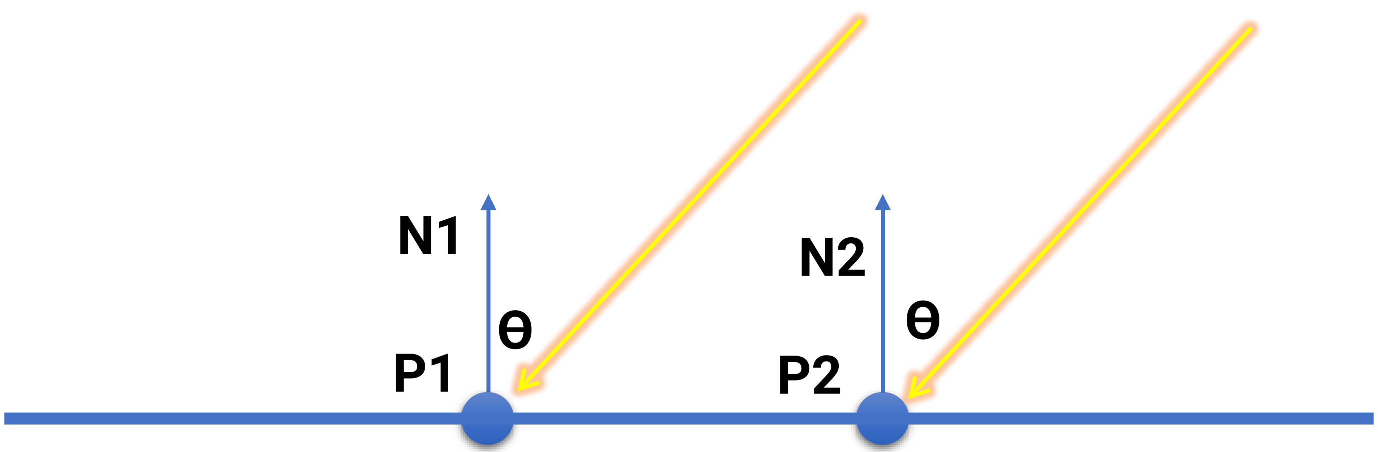
Area Light





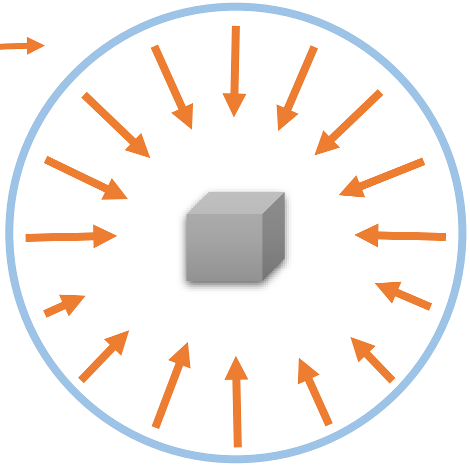
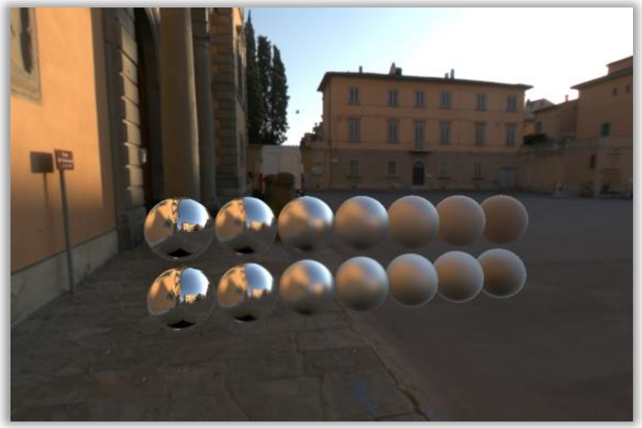
Distant Lights

- The distance between a light and a surface is long enough compared to the scene scale and **can be ignored**
  - **Lighting direction** is **fixed**
  - **No lighting attenuation**
- **Directional light (sun)** is the most common distant light



Environment Light

- Environment light illuminates the scene from a **virtual sphere at infinite distance**
- The spherical energy distribution is usually represented with longitude-latitude images
- Also called **image-based lighting (IBL)**

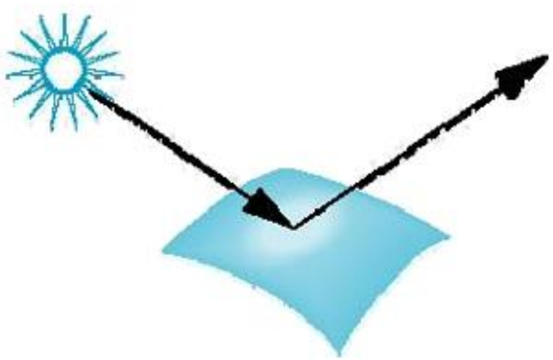




Environment Light (cont.)

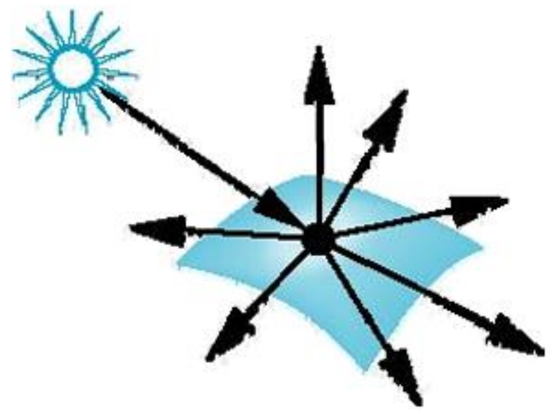
- Widely used in digital visual effects and film production

Materials

- **Surface types**
  - The **smoother** a surface, the more reflected light is concentrated in the direction a **perfect mirror** would reflect the light



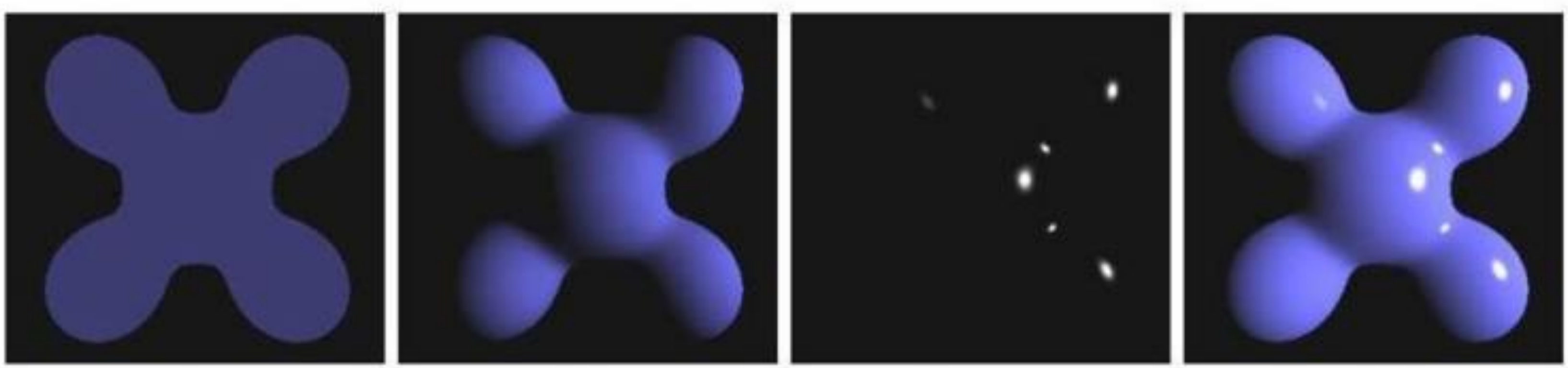
smooth surface



rough surface

Basics of Local Shading

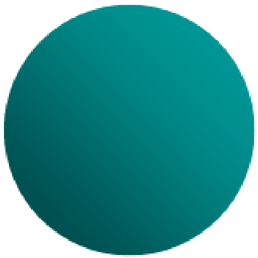
- **Diffuse reflection**
  - Light goes everywhere; colored by object color
- **Specular reflection**
  - Happens only near mirror configuration; usually white
- **Ambient reflection**
  - Constant accounted for other source of illumination

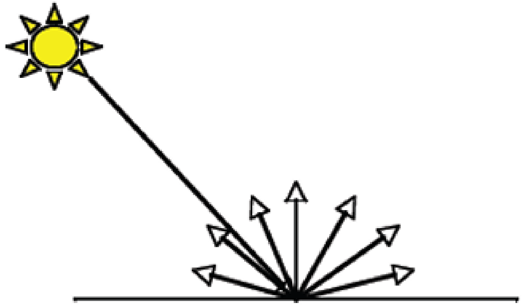


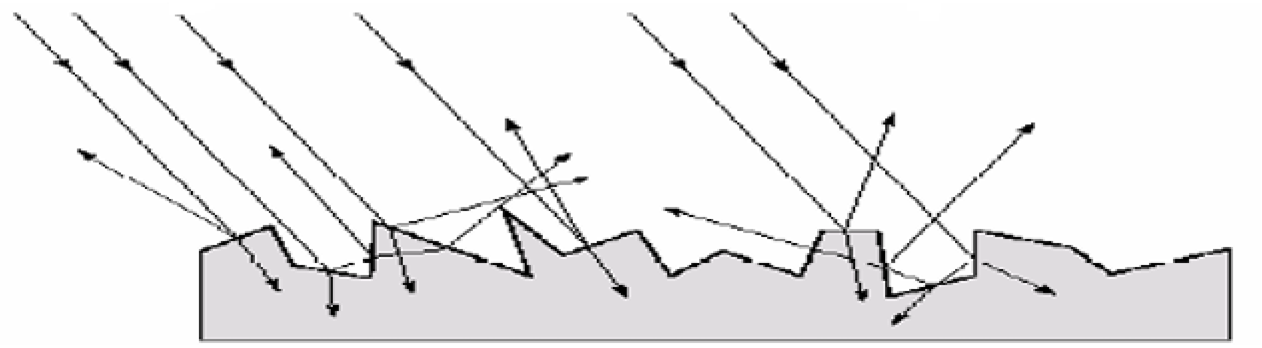
ambient diffuse specular

Diffuse Shading

- Assume light reflects **equally in all directions**
  - The surface is rough with lots of tiny microfacets
- Therefore, surface looks same color from all views (**view independent**)

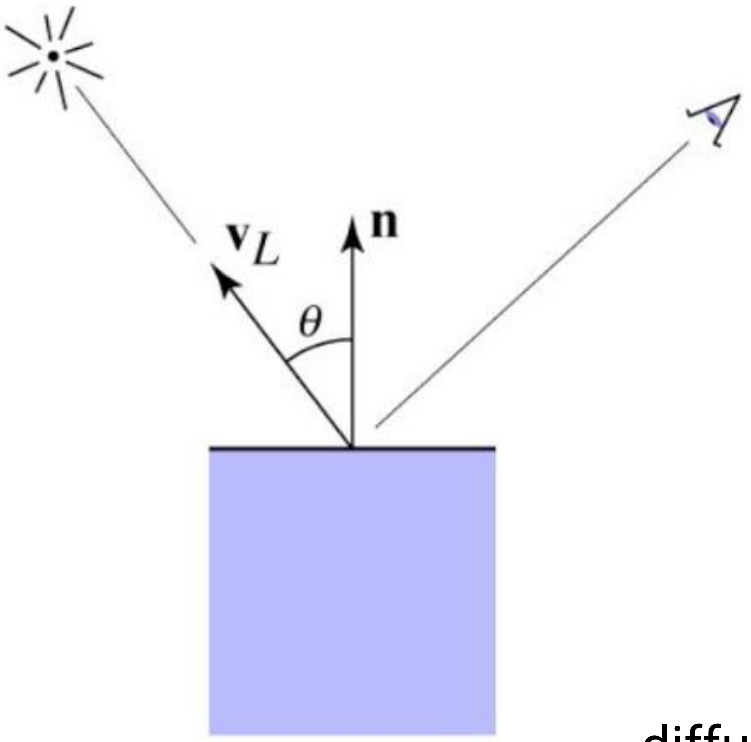






Diffuse Shading (cont.)

- Applies to diffuse, Lambertian or matte surface



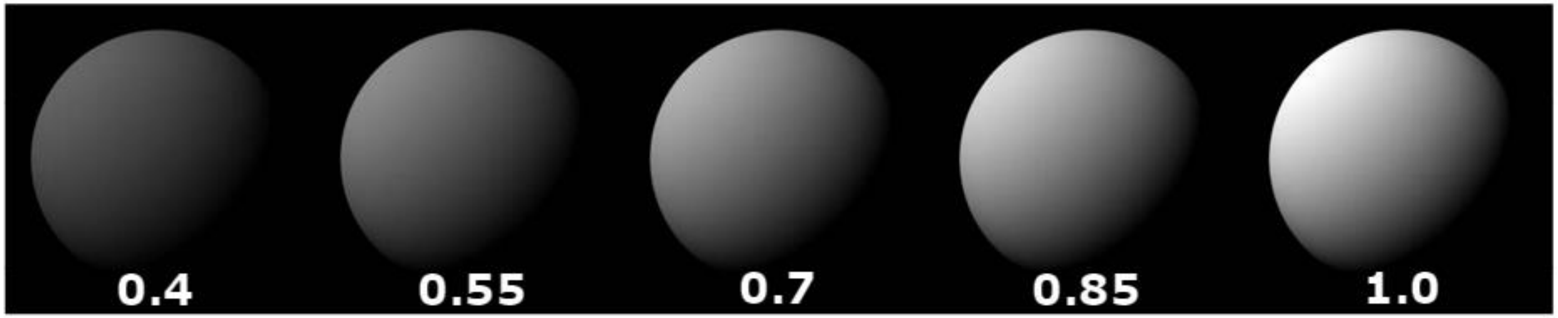
illumination from source

$$L_d = k_d \; I \; \max(0, n \cdot V_L)$$

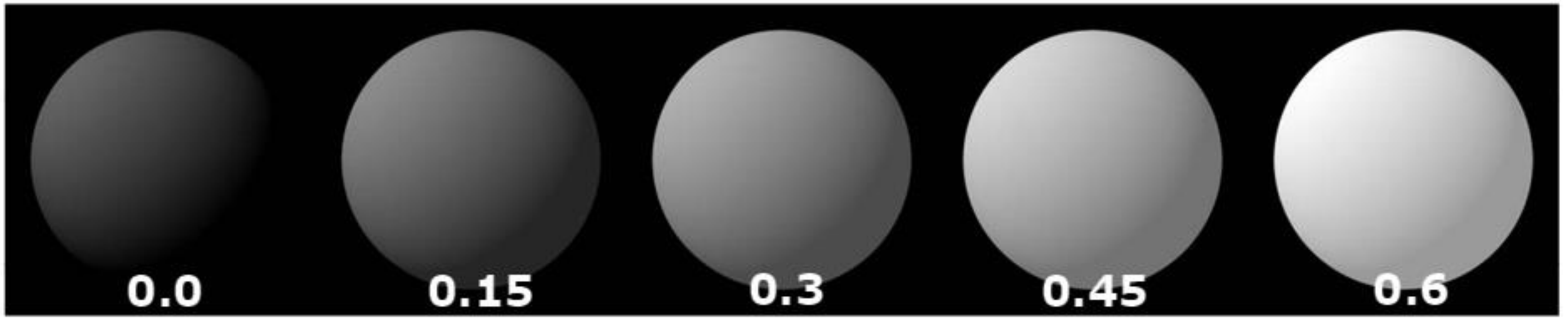
diffuse coefficient

diffusely reflected light

Diffuse Shading (cont.)



diffuse-reflection model with different k_d



ambient and diffuse-reflection model with different k_a

$I_a = 1.0 \quad k_d = 0.4$

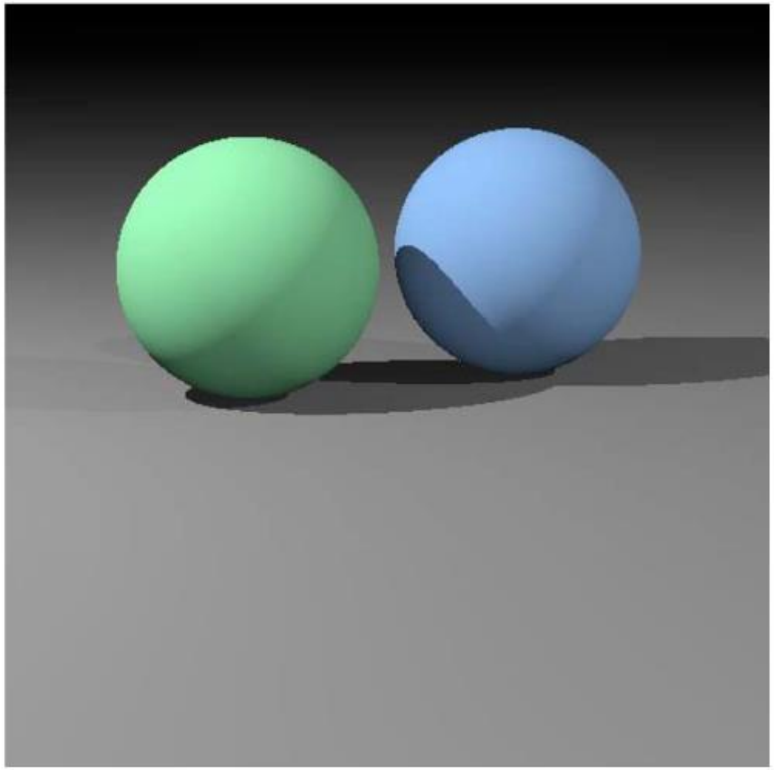
Diffuse Shading (cont.)

- For color objects, apply the formula for each color channel separately
- Light can also be non-white

Example:
white light: (0.9, 0.9, 0.9)
yellow light: (0.8, 0.8, 0.2)

$$L_d = \boxed{k_d}\,\boxed{I}\,\max(0, n \cdot V_L)$$

Example:
green ball: (0.2, 0.7, 0.2)
blue ball: (0.2, 0.2, 0.7)

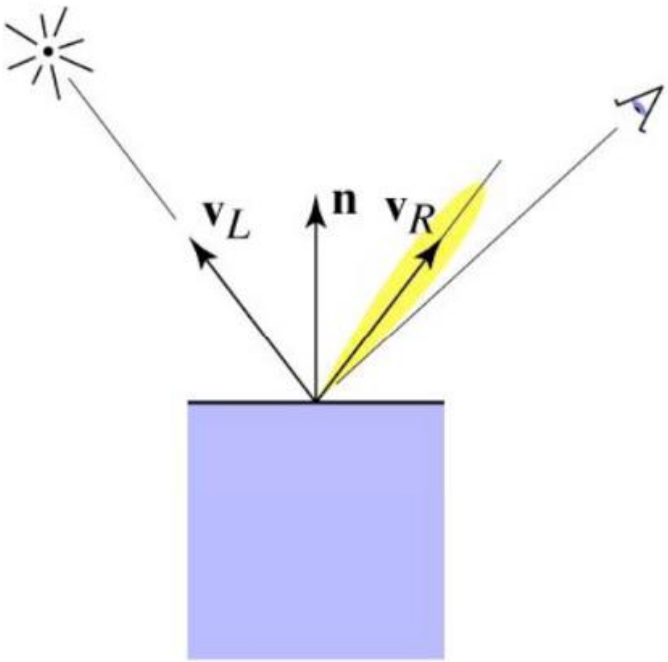


Specular Shading

- Some surfaces have highlights, mirror-like reflection
- **View direction dependent**
- Especially obvious for smooth shinny surfaces

Specular Shading (cont.)

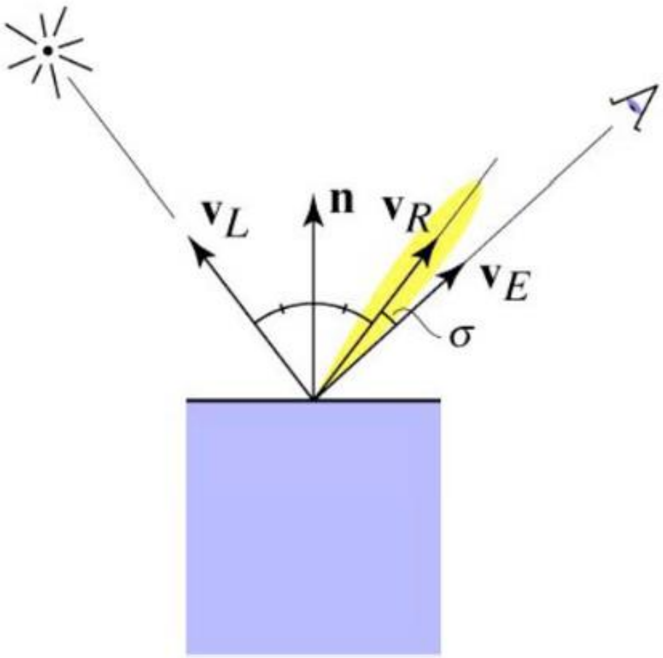
- Also known as glossy
- Phong specular model [1975]



$$
\begin{aligned}
V_R &= V_L + 2((\mathbf{n} \cdot V_L)\,\mathbf{n} - V_L) \\
&= 2(\mathbf{n} \cdot V_L)\,\mathbf{n} - V_L
\end{aligned}
$$

Specular Shading (cont.)

- Also known as glossy
- Phong specular model [1975]
    - Fall off gradually from the perfect reflection direction



$$V_R = V_L + 2((\mathbf{n} \cdot V_L)\,\mathbf{n} - V_L) \\ = 2(\mathbf{n} \cdot V_L)\,\mathbf{n} - V_L$$

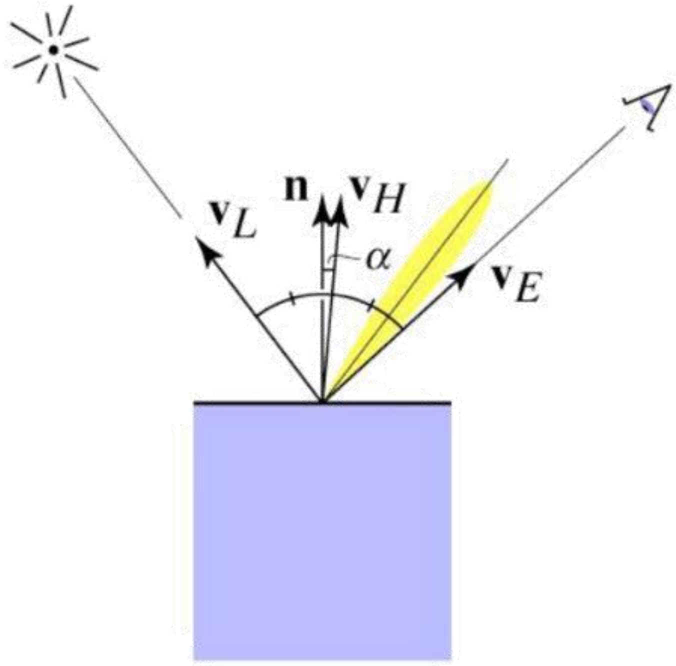
$$L_s = k_s\ I\ \max(0, \cos\sigma)^n \\ = k_s\ I\ \max(0, V_E \cdot V_R)^n$$

specular coefficient

specularly reflected light

Specular Shading (cont.)

- Phong variant: **Blinn-Phong**
  - Rather than computing reflection directly; just compare to normal bisection property
  - One can prove $\cos^n(\sigma) = \cos^{4n}(\alpha)$



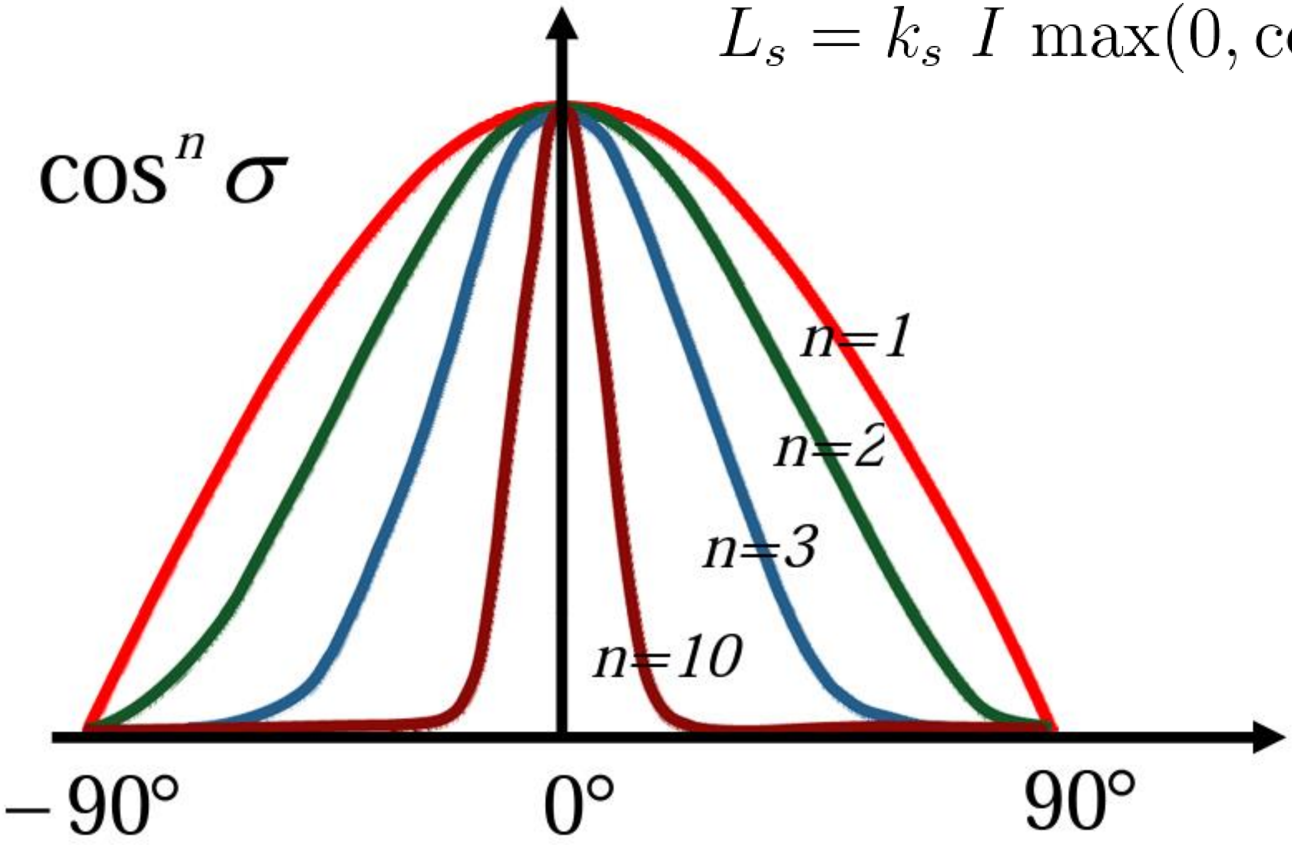
$$V_H = \text{bisector}(V_L, V_E) = \frac{(V_L + V_E)}{\|V_L + V_E\|}$$

$$L_s = k_s \; I \; \max(0, \cos\alpha)^n = k_s \; I \; \max(0, \mathbf{n} \cdot V_H)^n$$

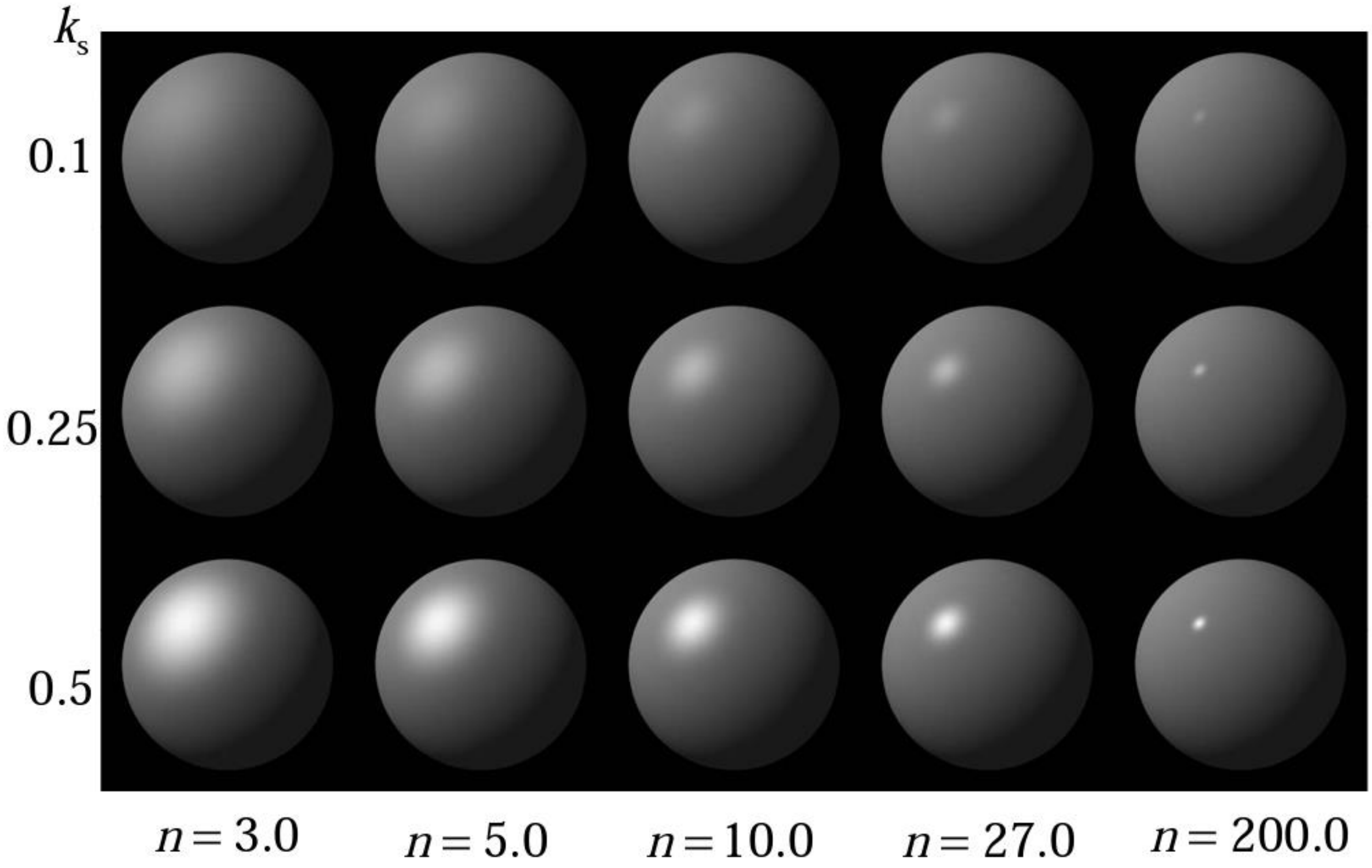
Specular Shading (cont.)

- Increase n narrows the lobe

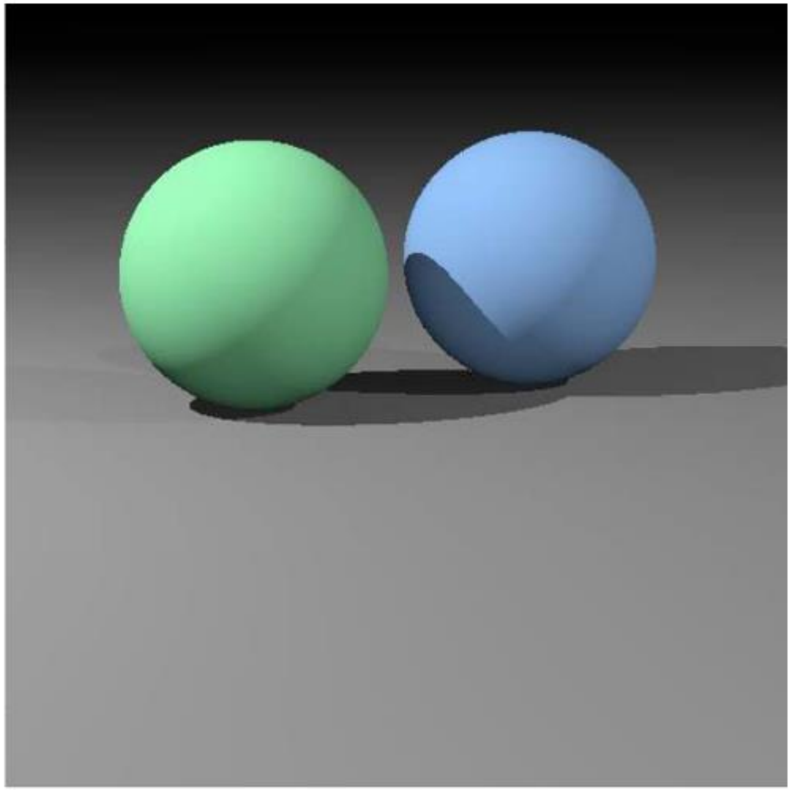
$$L_s = k_s \; I \; \max(0, \cos\alpha)^{n}$$



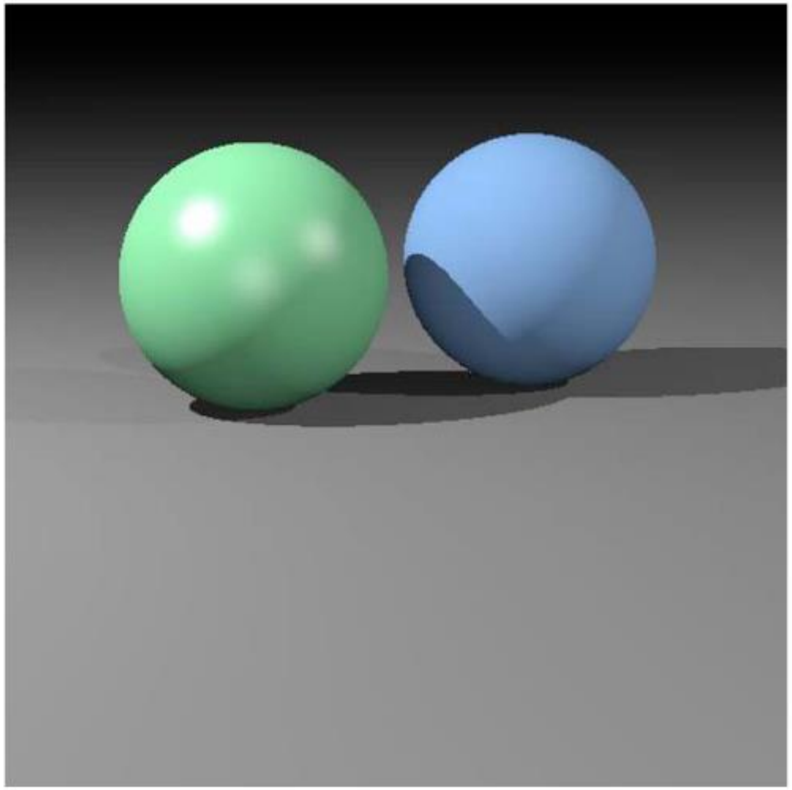
Specular Shading (cont.)



Specular Shading (cont.)



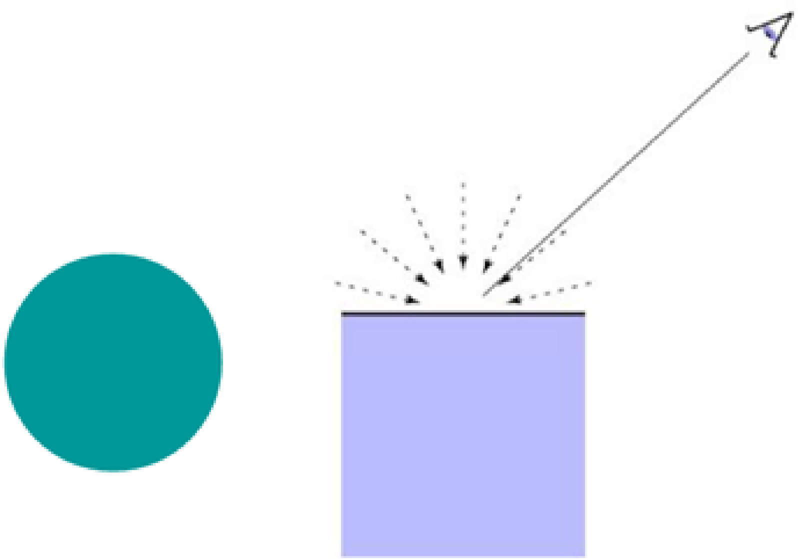
diffuse



diffuse + specular

Ambient Shading

- Add constant color to account for disregarded illumination and fill in black shadows
- A cheap hack



ambient light

$$L_a = k_a I_a$$

ambient coefficient

reflected ambient light

Put it All Together

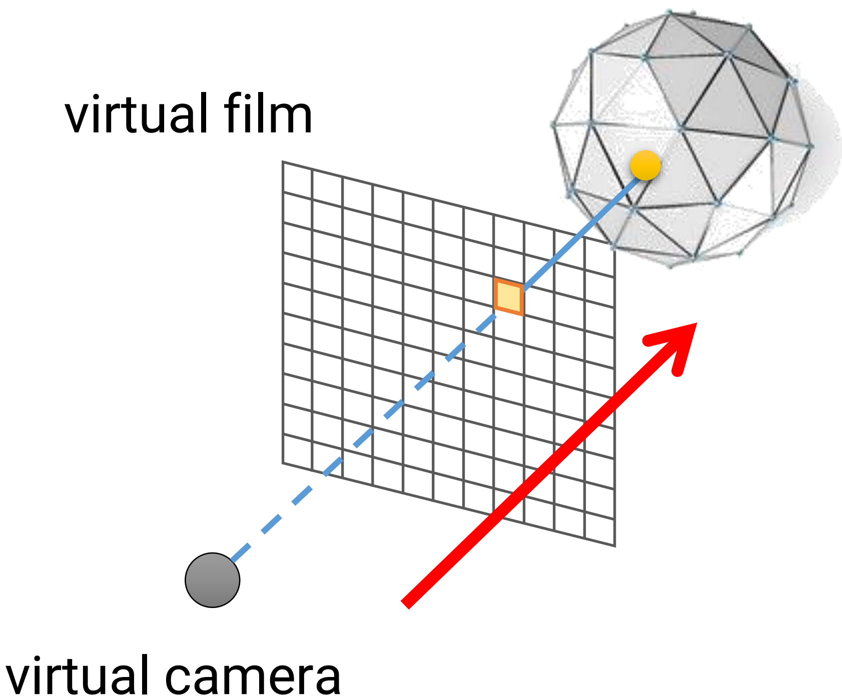
- Compute the contribution of a light to a point by including **ambient**, **diffuse**, and **specular** components

$$\begin{aligned} L &= L_a + L_d + L_s \\ &= k_a I_a + I(k_d \max(0, \mathbf{n} \cdot V_L) + k_s \max(0, \mathbf{n} \cdot V_H)^n) \end{aligned}$$

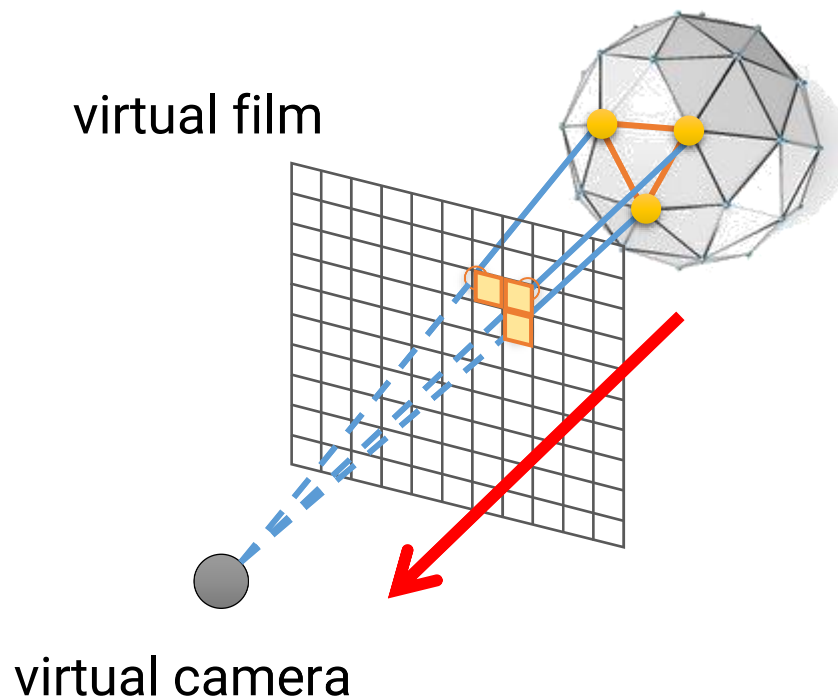
- If there are many lights, just sum over all the lights because lighting is **linear**

Bring Triangles into Pixels

Ray Tracing



Rasterization

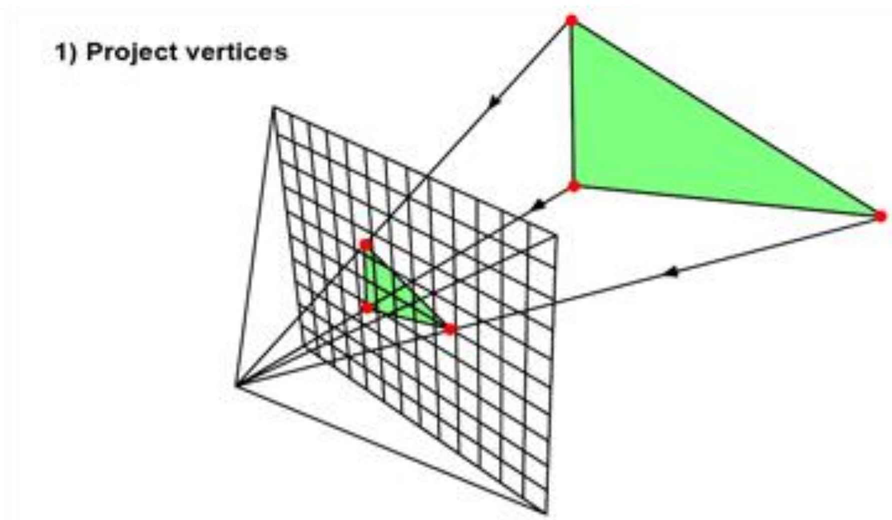


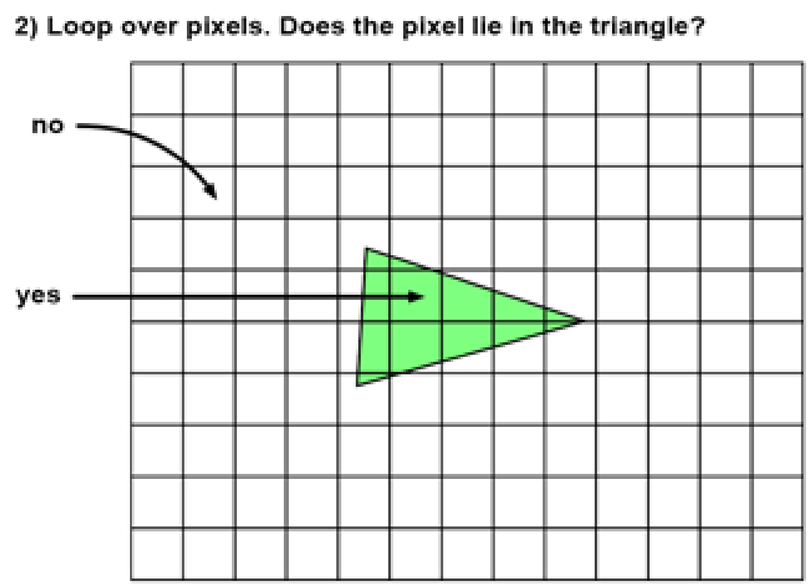
Graphics API (or Library)

- A program library designed to aid in rendering computer graphics to a monitor
- Typically involves providing optimized versions of functions that handle common **rendering tasks**
- **Rasterization-based**
- Common graphics APIs are
  - OpenGL
  - OpenGL ES
  - WebGL
  - DirectX
  - Metal
  - Vulkan

Rasterization

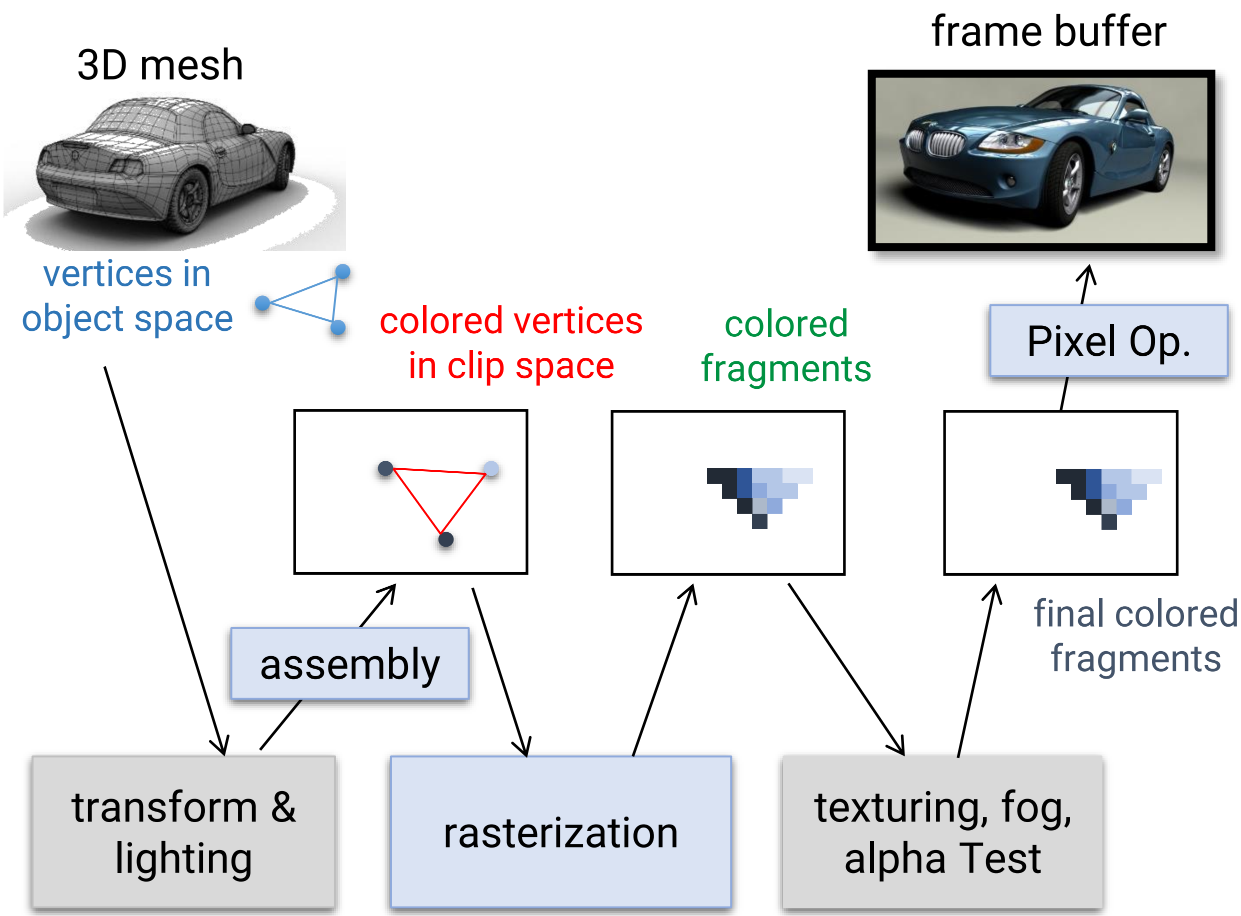
- Rasterization
  - Bring the **triangles** to **pixels**
  - Determine which pixels are covered by a projected 3D triangle



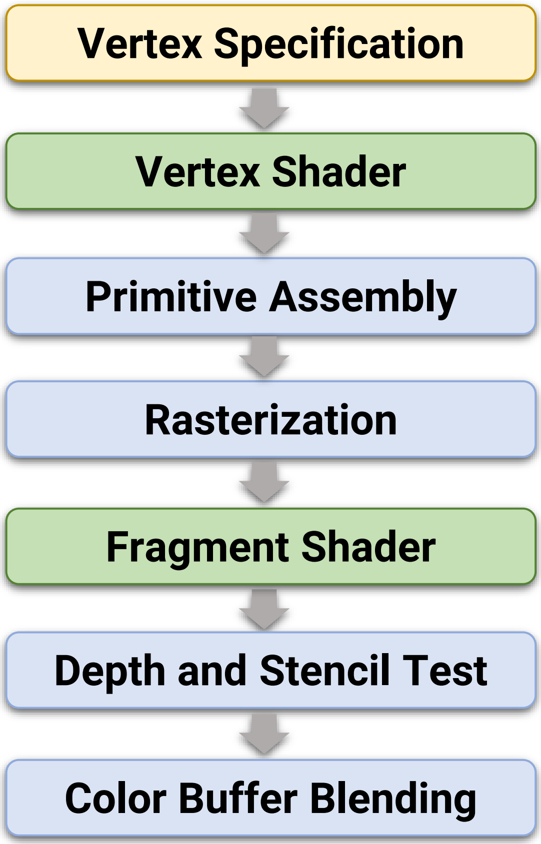


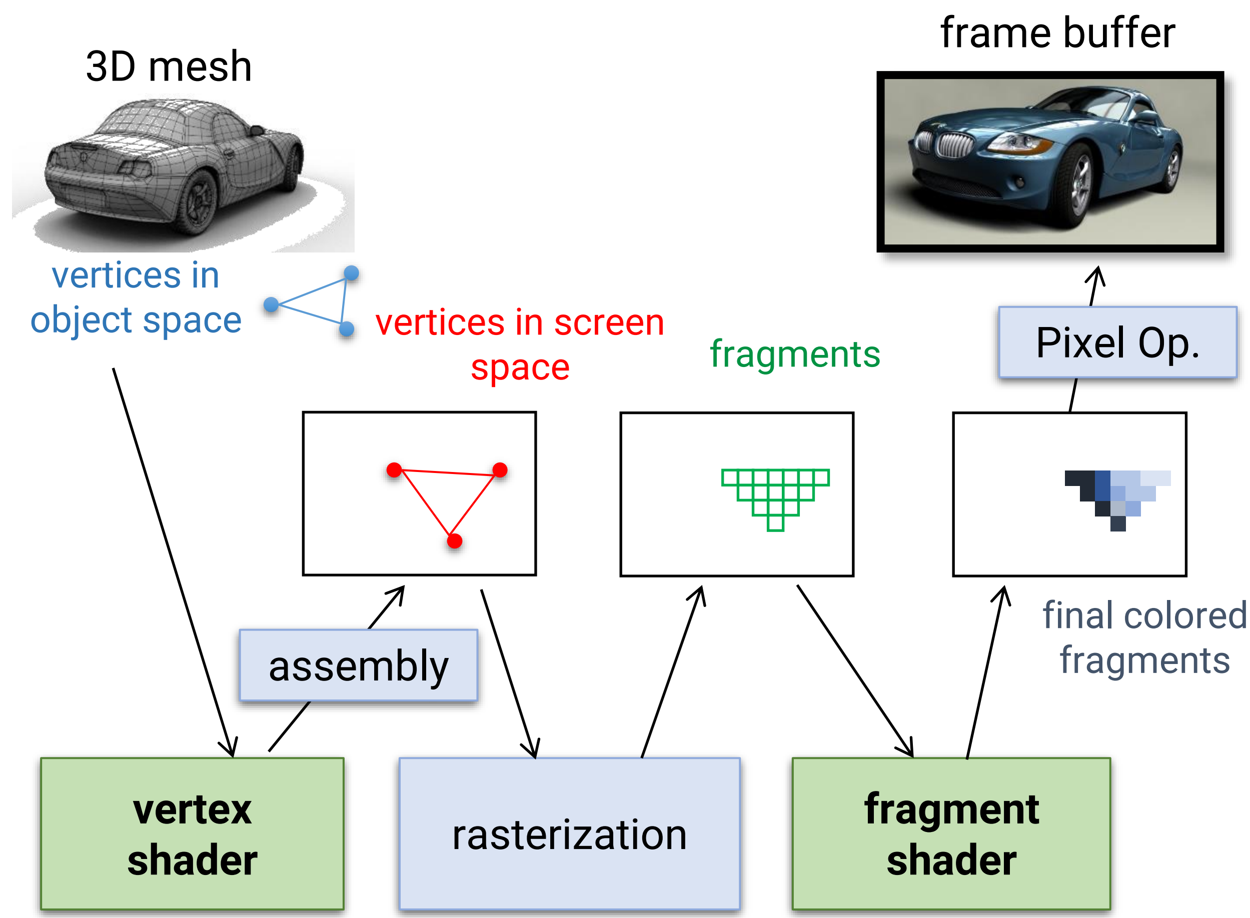
OpenGL 1.X (Fixed Function) Graphics Pipeline

- **Vertex Specification**
- **Transform and Lighting**
- **Primitive Assembly**
- **Rasterization**
- **Texturing and Fog**
- **Alpha Test**
- **Depth and Stencil Test**
- **Color Buffer Blending**



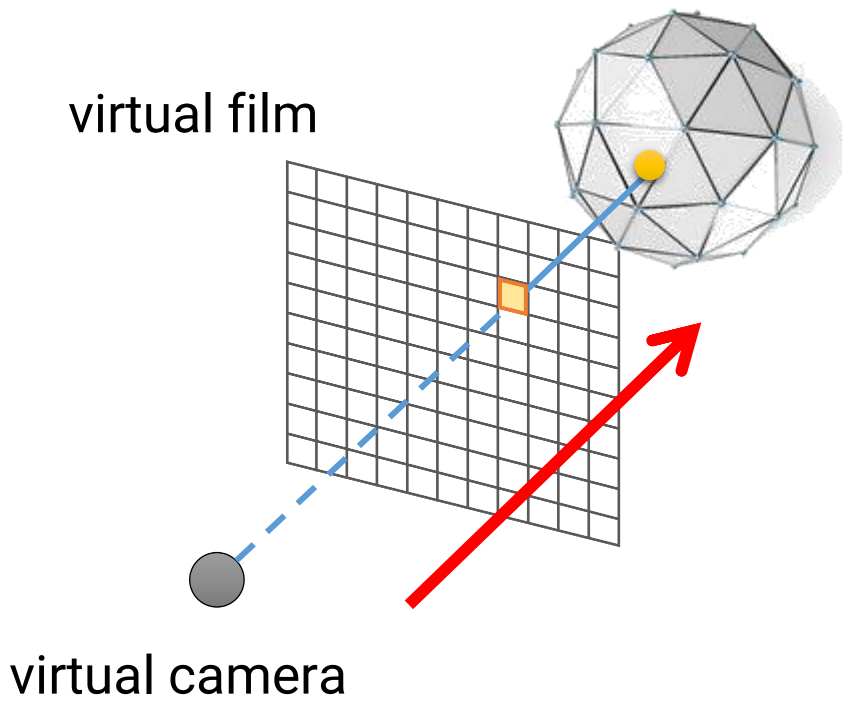
OpenGL 2.0 Graphics Pipeline



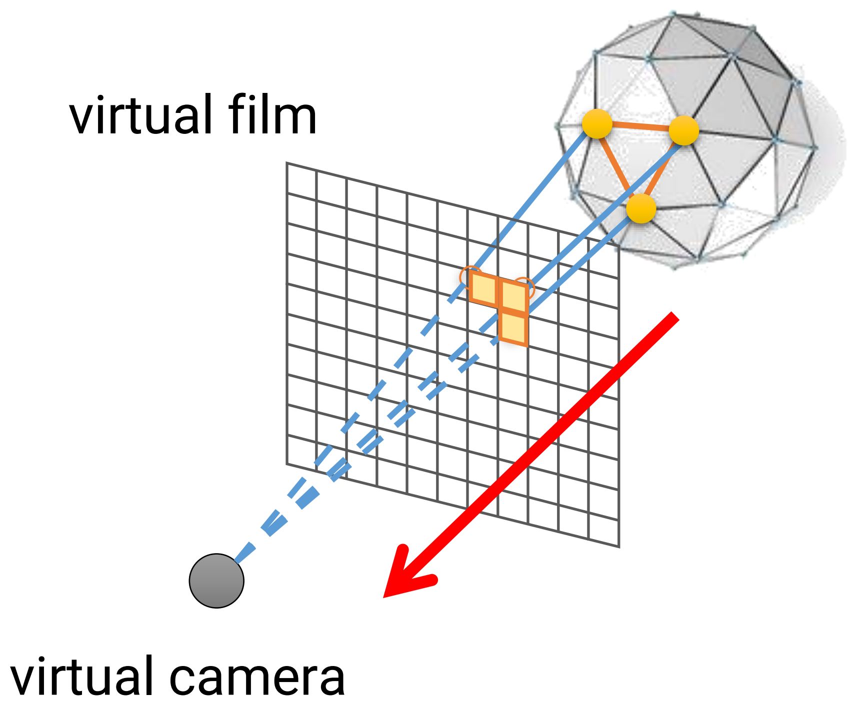


Bring Triangles into Pixels

Ray Tracing

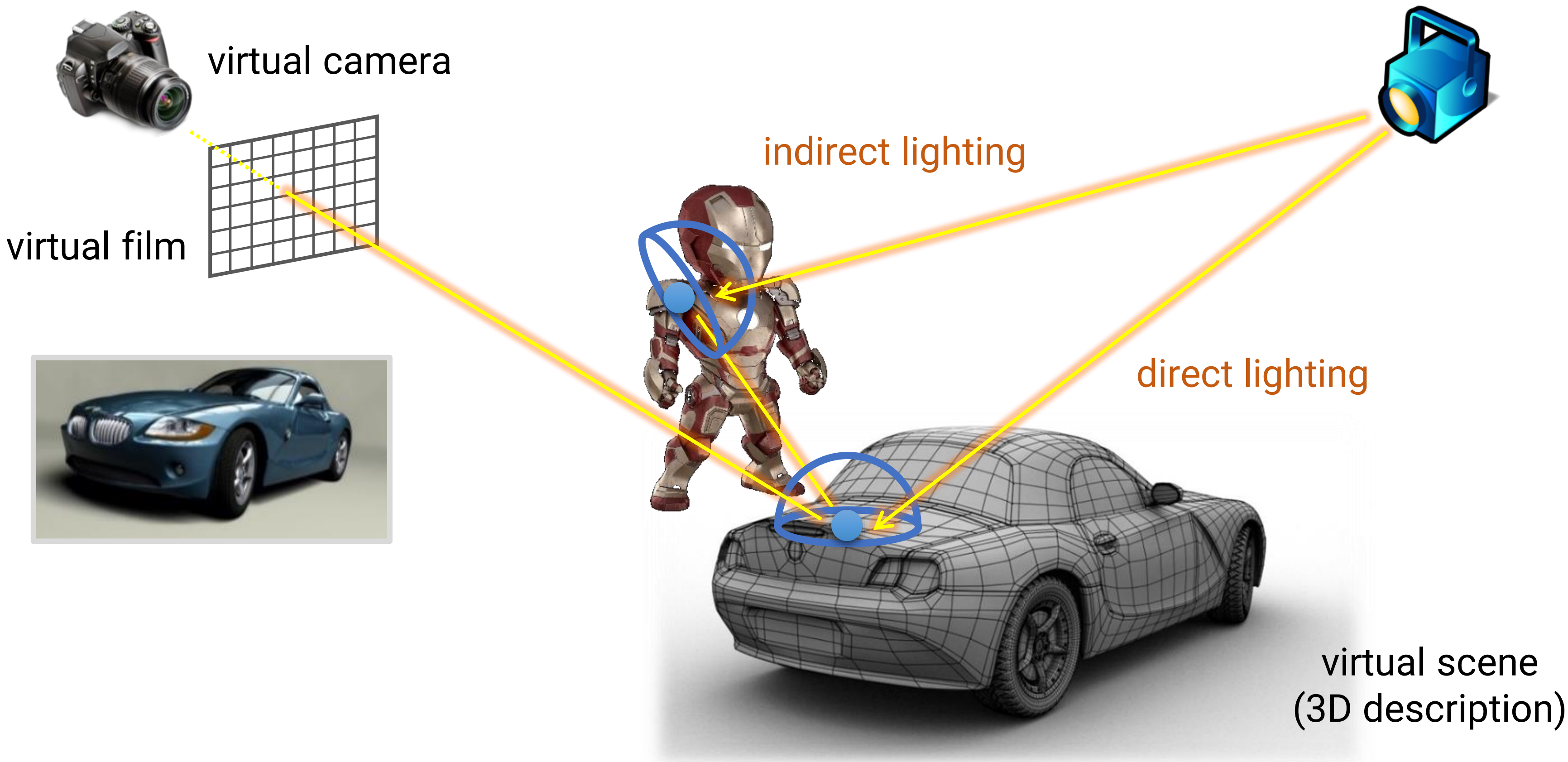


Rasterization



Ray Tracing

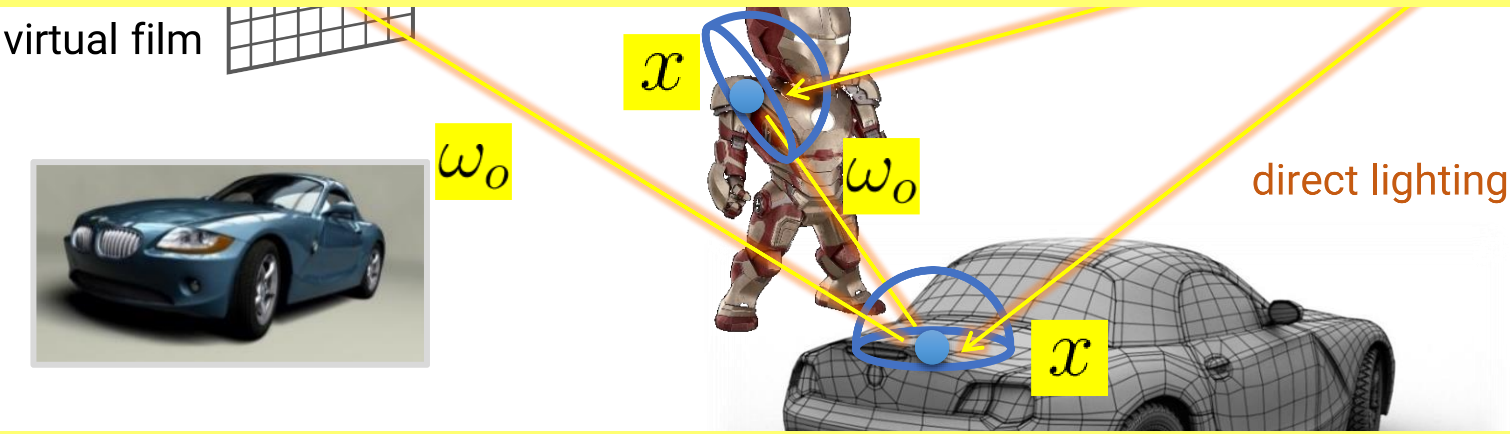
- Simulate a wide variety of light transport paths by tracing rays and calculate their carried energy



Ray Tracing

- Simulate a wide variety of light transport paths by tracing rays and calculate their carried energy

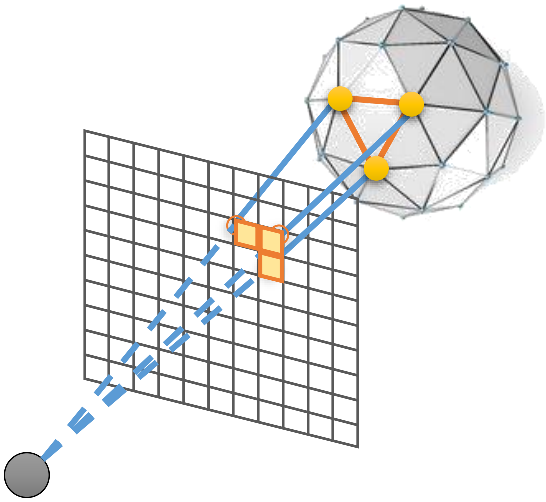
$$L(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega L_i(x,\omega_i) f_r(x,\omega_o \leftarrow \omega_i)(N(x)\cdot\omega_i)d\omega_i$$

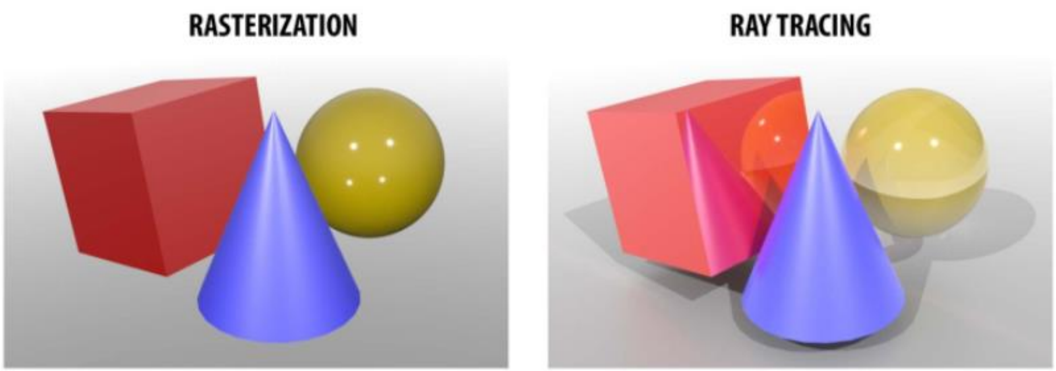


$$L(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega L_i(x,\omega_i) f_r(x,\omega_o \leftarrow \omega_i)(N(x)\cdot\omega_i)d\omega_i$$

Rasterization v.s. Ray Tracing

- Rasterization is more friendly to hardware and usually has higher parallelism
- But it is more difficult to simulate effects such as reflection, refraction, shadows, and global illumination
  - Need specialized algorithms

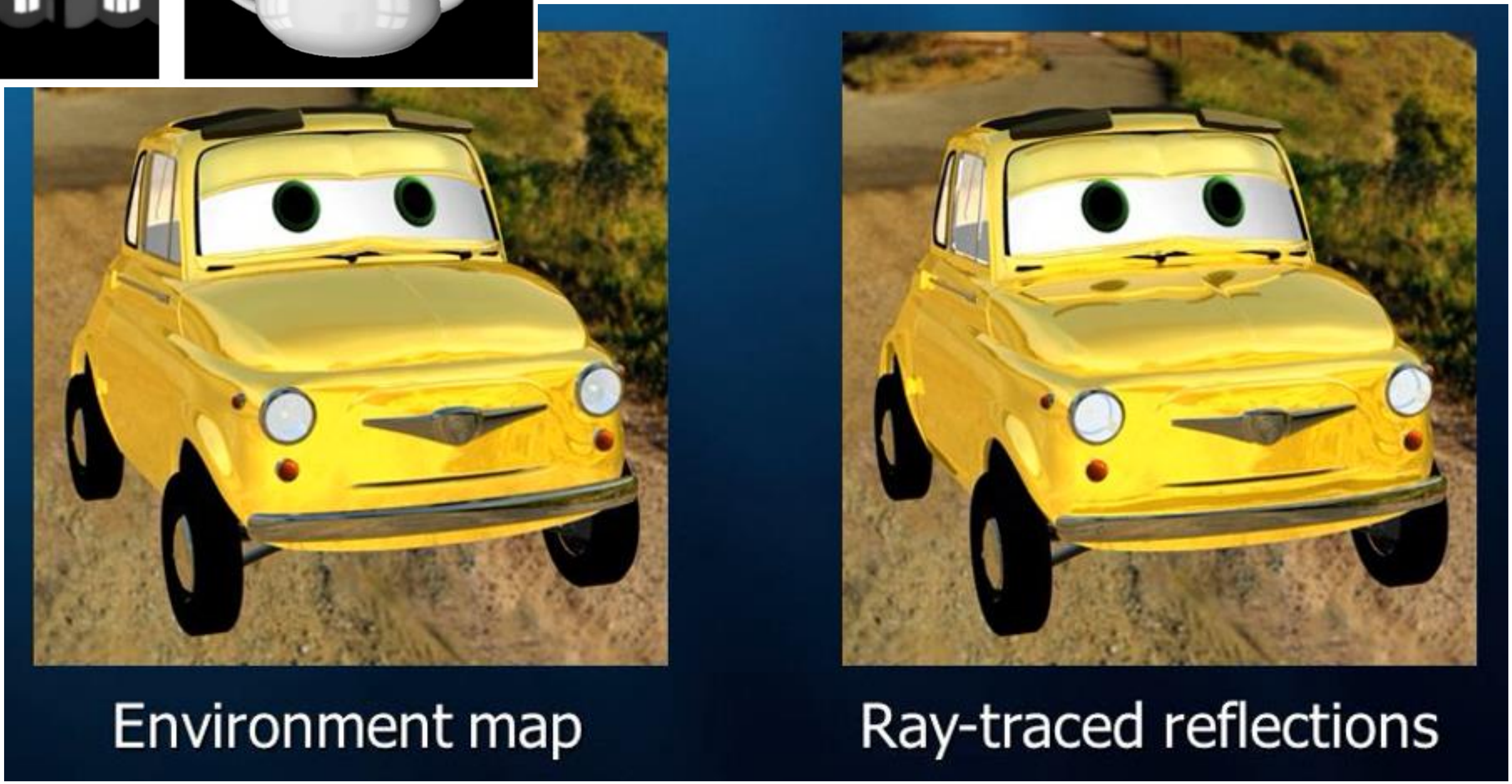




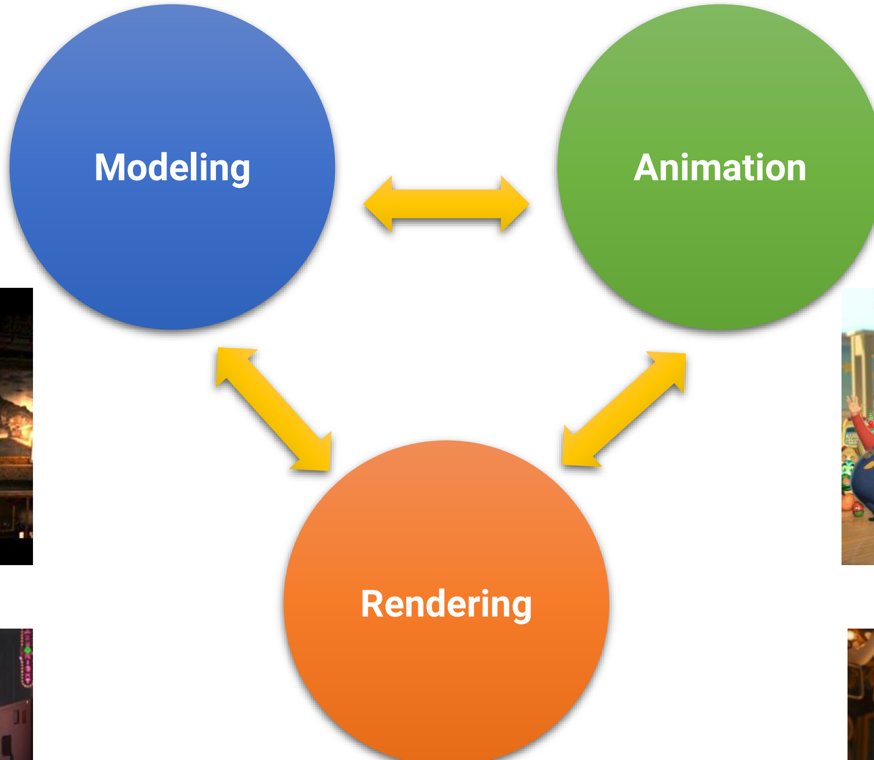
Rasterization v.s. Ray Tracing

- Ray tracing is more general
- However, its simulator usually has a slow convergence rate and produces lots of noises when samples are not enough

Rasterization v.s. Ray Tracing

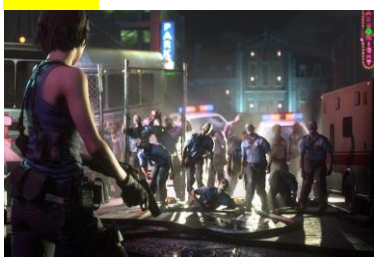


Real-time v.s. Offline Graphics



1999

2020



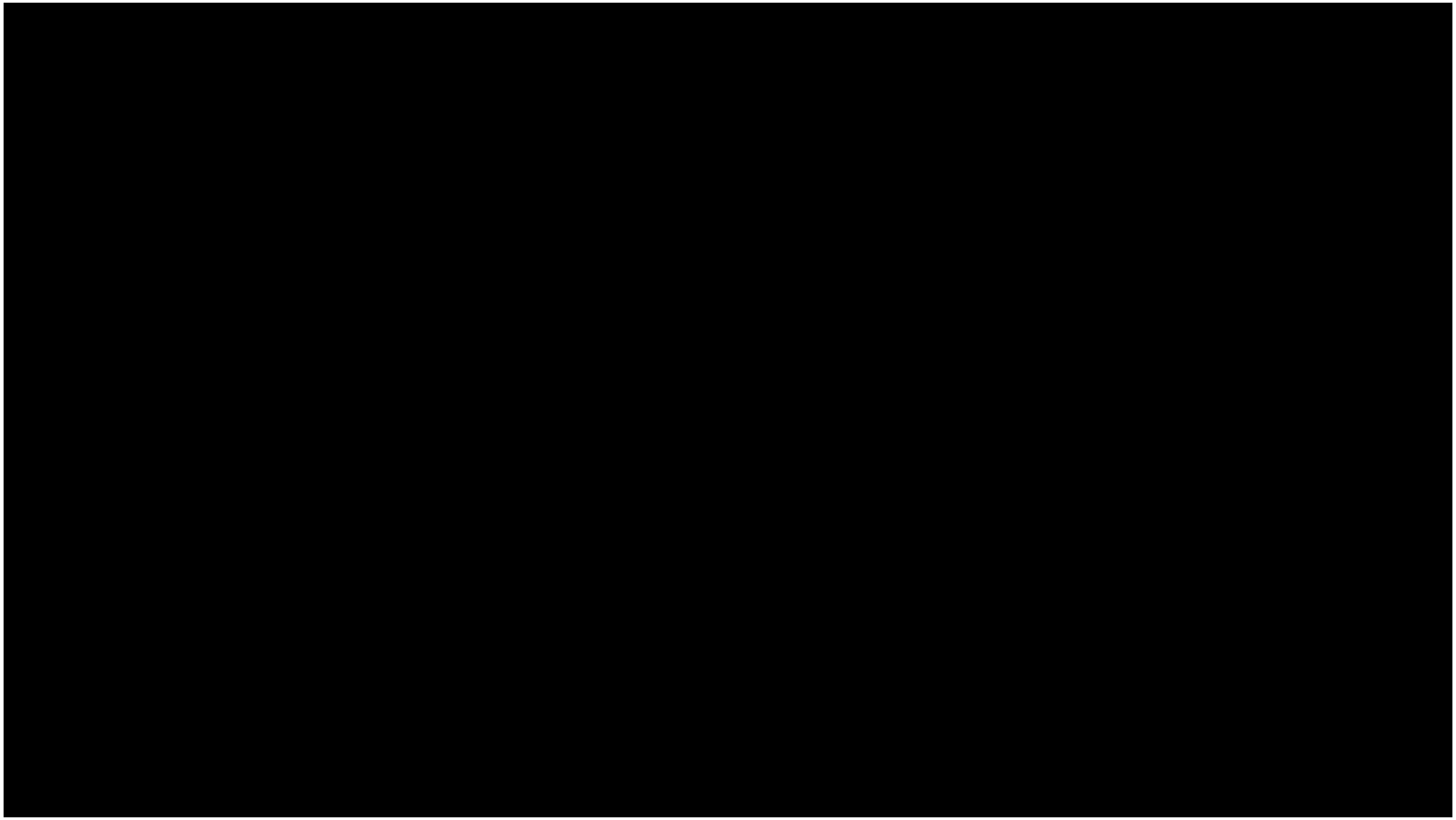
1999

2019

real-time ⟷ **offline**

Real-time Ray Tracing

- FIRST DAY: A Star Wars short film made with UE5



How to Learn 3D Computer Graphics?

- Online materials
  - https://ogldev.org/
  - http://www.opengl-tutorial.org/
  - https://learnopengl.com/
  - https://antongerdelan.net/opengl/
- Or …
- Come to my class ***“Introduction to Computer Graphics”*** **next semester**!